YEAR 2022-23

CBSE SAMPLE QUESTION PAPER

MATHEMATICS (Standard)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 5 marks, 2 Questions of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF $(a, b) = p^mq^n$ and LCM$(a, b) = p^rq^s$, then $(m + n)(r + s)$ =

(a) 15 (b) 30 (c) 35 (d) 72

Ans. (c) 35 1

2. Let p be a prime number. The quadratic equation having its roots as factors of p is

(a) $x^2 - px + p = 0$ (b) $x^2 - (p + 1)x + p = 0$

(c) $x^2 + (p + 1)x + p = 0$ (d) $x^2 - px + p + 1 = 0$

Ans. (b) $x^2 - (p + 1)x + p = 0$ 1

3. If α and β are the zeroes of a polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then p is

(a) –2/3 (b) 2/3 (c) 1/3 (d) –1/3

Ans. (b) 2/3 1

4. If three system of equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then k =

(a) –1 (b) 0 (c) 1 (d) 2

Ans. (d) 2 1

5. If the vertices of a parallelogram PQRS, taken in order, are P(3,4), Q(–2,3) and R(–3, –2), then the coordinates of its fourth vertex S are

(a) (–2, –1) (b) (–2, –3) (c) (2, –1) (d) (1, 2)

Ans. (c) (2, –1) 1

6. If $\Delta ABC \sim \Delta PQR$, AM and PN are altitudes of ΔABC and ΔPQR respectively and $AB^2 : PQ^2 = 4 : 9$, then AM : PN =

(a) 3 : 2 (b) 16 : 81 (c) 4 : 9 (d) 2 : 3

Ans. (d) 2 : 3 1

7. If $x \tan 60° \cos 60° = \sin 60° \cot 60°$, then x =

(a) cos 30° (b) tan 30° (c) sin 30° (d) cot 30°

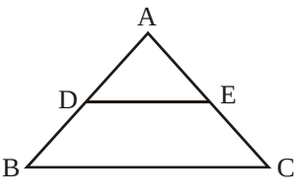
Ans. (b) tan 30° 1

8. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta$ =

(a) 1 (b) 2 (c) 3 (d) 4

Ans. (b) 2 1

9. In the given figure, DE || BC, AE = a units, EC = b units, DE = x units and BC = y units. Which of the following is true?



(a) $x = \frac{a+b}{ay}$ (b) $y = \frac{ax}{a+b}$ (c) $x = \frac{ay}{a+b}$ (d) $\frac{x}{y} = \frac{a}{b}$

Ans. (c) $x = \frac{ay}{a+b}$ 1

10. ABCD is a trapezium with AD || BC and AD = 4 cm. If the diagonals AC and BD intersect each other at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then BC =

(a) 6 cm (b) 7 cm (c) 8 cm (d) 9 cm

Ans. (c) 8 cm 1

11. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to

(a) $\frac{3\sqrt{3}}{2}$ cm (b) 3 cm (c) 6 cm (d) $3\sqrt{3}$ cm

Ans. (d) $3\sqrt{3}$ cm 1

12. The area of the circle that can be inscribed in a square of side 6 cm is

(a) 36π cm^2 (b) 18π cm^2 (c) 12π cm^2 (d) 9π cm^2

Ans. (d) 9π cm^2 1

13. The sum of the length, breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm. The total surface area of the cuboid is

(a) 48 cm^2 (b) 72 cm^2 (c) 96 cm^2 (d) 108 cm^2

Ans. (c) 96 cm^2 1

14. If the difference of Mode and Median of a data is 24, then the difference of median and mean is

(a) 8 (b) 12 (c) 24 (d) 36

Ans. (b) 12 1

15. The number of revolutions made by a circular wheel of radius 0.25 m in rolling a distance of 11 km is

(a) 2800 (b) 4000 (c) 5500 (d) 7000

Ans. (d) 7000 1

16. For the following distribution,

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

the sum of the lower limits of the median and modal class is

(a) 15 (b) 25 (c) 30 (d) 35

Ans. (b) 25 1

17. Two dice are rolled simultaneously. What is the probability that 6 will come up on at least one dice?

(*a*) 1/6 (*b*) 7/36 (*c*) 11/36 (*d*) 13/36

Ans. (*c*) 11/36 1

18. If 5 tan β = 4, then $\frac{5 \sin \beta - 2 \cos \beta}{5 \sin \beta + 2 \cos \beta}$ =

(*a*) 1/3 (*b*) 2/5 (*c*) 3/5 (*d*) 6

Ans. (*a*) 1/3 1

DIRECTION : In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option.

19. Statement A (Assertion) : If product of two numbers is 5780 and their HCF is 17, then their LCM is 340.

Statement R (Reason) : HCF is always a factor of LCM.

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

Ans. (*b*) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A). 1

20. Statement A (Assertion) : If the co-ordinates of the mid-points of the sides AB and AC of ΔABC are D(3, 5) and E(–3, –3) respectively, then BC = 20 units.

Statement R (Reason) : The line-segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

Ans. (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). 1

SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. If 49*x* + 51*y* = 499, 51*x* + 49*y* = 501, then find the values of *x* and *y*.

Sol. Adding the two equations and dividing by 100, we get :

$x + y = 10.$ ½

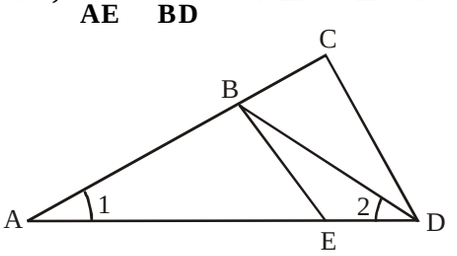
Subtracting the two equations and dividing by –2, we get :

$x - y = 1$ ½

Solving these two new equations, we get, $x = \frac{11}{2}$, ½

$y = \frac{9}{2}$ ½

22. In the figure given below, $\frac{AD}{AE} = \frac{AC}{BD}$ and $\angle 1 = \angle 2$. Show that $\Delta BAE \sim \Delta CAD$.



Sol. In ΔABD, $\angle 1 = \angle 2$

$\therefore$ $AB = BD$ ...(*i*) ½

Given, $\frac{AD}{AE} = \frac{AC}{BD}$

Using equation (*i*), we get

$\frac{AD}{AE} = \frac{AC}{AB}$ ...(*ii*) ½

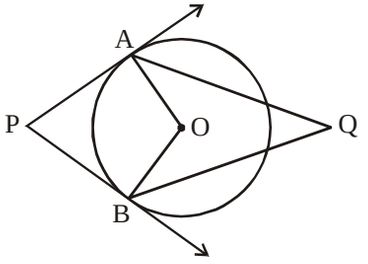
In ΔBAE and ΔCAD, by equation (*ii*),

$\frac{AC}{AB} = \frac{AD}{AE}$ ½

$\angle A = \angle A$ (Common)

$\therefore$ $\Delta BAE = \Delta CAD$ [By SAS similarity criterion] ½

23. In the given figure, O is the centre of the circle. Find $\angle AQB$, given that PA and PB are tangents to the circle and $\angle APB = 75°$.



Sol. $\angle PAO = \angle PBO = 90°$ (Angle between radius and tangent) ½

and $\angle APB = 75°$ (Given)

$\angle AOB = 105°$ (By angle sum property of a quadrilateral) ½

$\angle AQB = \frac{1}{2} 105° = 52.5°$ (Angle at the remaining part of the circle is half the angle subtended by the arc at the centre) 1

24. The length of the minute hand of a clock is 6 cm. Find the area swept by it when it moves from 7 : 05 p.m. to 7 : 40 p.m.

Sol. We know that, in 60 minutes, the tip of minute hand moves 360°.

In 1 minute, it will move = $\frac{360°}{60} = 6°$ ½

$\therefore$ From 7 : 05 pm to 7: 40 pm *i.e.*, 35 min., it will move through = $35 \times 6° = 210°$ ½

$\therefore$ Area of swept by the minute hand in 35 min = Area of sector with central angle θ of 210° and radius of 6 cm

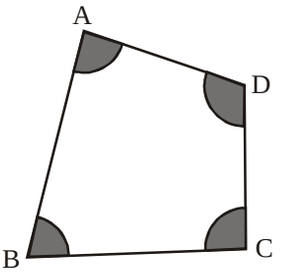
$= \frac{210}{360} \times \pi \times 6^2$ ½

$= \frac{7}{12} \times \frac{22}{7} \times 6 \times 6$

$= 66 \text{ cm}^2$ ½

Or

In the given figure, arcs have been drawn of radius 7 cm each with vertices A, B, C and D of a quadrilateral ABCD as centres. Find the area of the shaded region.



Sol. Let the measure of ∠A, ∠B, ∠C and ∠D be θ_1, θ_2, θ_3 and θ_4 respectively.

Required area = Area of sector with centre A
+ Area of sector with centre B
+ Area of sector with centre C
+ Area of sector with centre D ½

$= \frac{\theta_1}{360} \times \pi \times 7^2 + \frac{\theta_2}{360} \times \pi \times 7^2 + \frac{\theta_3}{360} \times \pi \times 7^2 + \frac{\theta_4}{360} \times \pi \times 7^2$ ½

$= \frac{(\theta_1 + \theta_2 + \theta_3 + \theta_4)}{360} \times \pi \times 7^2$

$= \frac{(360)}{360} \times \frac{22}{7} \times 7 \times 7$ (By angle sum property of a quadrilateral) ½

$= 154 \text{ cm}^2$ ½

25. If sin (A + B) = 1 and cos (A – B) = $\frac{\sqrt{3}}{2}$, 0° < A + B ≤ 90° and A > B, then find the measures of angles A and B.

Sol. sin (A + B) = 1 = sin 90°, so A + B = 90° ...(*i*) ½

cos (A – B) = $\frac{\sqrt{3}}{2}$ = cos 30°, so A – B = 30° ...(*ii*) ½

From (*i*) and (*ii*), ∠A = 60° ½

and ∠B = 30° ½

Or

Find an acute angle θ, when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$.

Sol. $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

Dividing the numerator and denominator of LHS by cos θ, we get ½

$\frac{1 - \tan\theta}{1 + \tan\theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$ ½

Which, on simplification (or comparison), gives $\tan\theta = \sqrt{3}$ ½

or $\theta = 60°$ ½

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Given that $\sqrt{3}$ is irrational, prove that $5 + 2\sqrt{3}$ is irrational.

Sol. Let us assume $5 + 2\sqrt{3}$ is rational. Then it must be in the form of $\frac{p}{q}$, where p and q are co-prime integers and $q \neq 0$ 1

i.e., $5 + 2\sqrt{3} = \frac{p}{q}$ ½

So, $\sqrt{3} = \frac{p - 5q}{2q}$ ...(*i*) ½

Since p, q, 5 and 2 are integers and $q \neq 0$, RHS of equation (i) is rational. But LHS of (i) is $\sqrt{3}$ which is irrational. This is not possible. ½

This contradiction has arisen due to our wrong assumption that $5 + 2\sqrt{3}$ is rational. So, $5 + 2\sqrt{3}$ is irrational. ½

27. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of the polynomial $2x^2 - 5x - 3$, then find the values of p and q.

Sol. Let α and β be the zeroes of the polynomial $2x^2 - 5x - 3$.

Then, $\alpha + \beta = \frac{5}{2}$ ½

and $\alpha\beta = \frac{-3}{2}$ ½

Let 2α and 2β be the zeroes $x^2 + px + q$.

Then, $2\alpha + 2\beta = -p$ ½

$\Rightarrow$ $2(\alpha + \beta) = -p$

$2 \times \frac{5}{2} = -p$

So, $p = -5$ ½

and $2\alpha \times 2\beta = q$ ½

$4\alpha\beta = q$

So, $q = 4 \times \frac{-3}{2} = -6$ ½

28. A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr; it would have taken 6 hours more than the scheduled time. Find the length of the journey.

Sol. Let the actual speed of the train be x km/hr and let the actual time taken be y hours.

Distance covered is xy km. ½

If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours *i.e.*, when speed is $(x + 6)$ km/hr, time of journey is $(y - 4)$ hours.

$\therefore$ Distance covered $= (x + 6)(y - 4)$

$\Rightarrow$ $xy = (x + 6)(y - 4)$

$\Rightarrow$ $-4x + 6y - 24 = 0$ ½

$\Rightarrow$ $-2x + 3y - 12 = 0$ ...(i)

Similarly, $xy = (x - 6)(y + 6)$

$\Rightarrow$ $6x - 6y - 36 = 0$

$\Rightarrow$ $x - y - 6 = 0$ ...(ii) ½

Solving (i) and (ii) we get $x = 30$ and $y = 24$ 1

Putting the values of x and y in equation (i), we obtain

Distance $= (30 \times 24)$ km $= 720$ km.

Hence, the length of the journey is 720 km. ½

Or

Anuj had some chocolates, and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 chocolates and the second lot at the rate of ₹ 1 per chocolate, and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per chocolate, and the second lot at the rate of ₹ 4 for 5 chocolates, his total collection would have been ₹ 460.

Find the total number of chocolates he had.

Sol. Let the number of chocolates in lot A be x. ½

And let the number of chocolates in lot B be y.

$\therefore$ Total number of chocolates $= x + y$.

Price of 1 chocolate = ₹ $\frac{2}{3}$, so for x chocolates = ₹ $\frac{2}{3}x$

and price of y chocolates at the rate of ₹ 1 per chocolate = y.

$\therefore$ By the given condition, $\frac{2}{3}x + y = 400$

$\Rightarrow$ $2x + 3y = 1200$ ...(i) ½

Similarly, $x + \frac{4}{5}y = 460$

$\Rightarrow$ $5x + 4y = 2300$ ...(ii) ½

Solving (i) and (ii), we get

$x = 300$ and $y = 200$

$\therefore$ $x + y = 300 + 200 = 500$ 1

So, Anuj had 500 chocolates. ½

29. Prove that : $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\,\text{cosec}\,\theta - 2\sin\theta\cos\theta$.

Sol. LHS : $\frac{\frac{\sin^3\theta}{\cos^3\theta}}{\frac{1+\sin^2\theta}{\cos^2\theta}} + \frac{\frac{\cos^3\theta}{\sin^3\theta}}{\frac{1+\cos^2\theta}{\sin^2\theta}}$ ½

$= \frac{\frac{\sin^3\theta}{\cos^3\theta}}{\frac{(\cos^2\theta+\sin^2\theta)}{\cos^2\theta}} + \frac{\frac{\cos^3\theta}{\sin^3\theta}}{\frac{(\sin^2\theta+\cos^2\theta)}{\sin^2\theta}}$

$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$ ½

$= \frac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta}$ ½

$= \frac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta.\cos^2\theta}{\cos\theta.\sin\theta}$ ½

$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$ ½

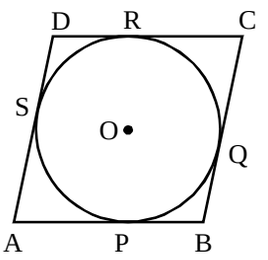
$= \frac{1}{\cos\theta\sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \sec\theta\,\text{cosec}\,\theta - 2\sin\theta\cos\theta$

$= \text{RHS}$ ½

30. Prove that a parallelogram circumscribing a circle is a rhombus.

Sol.



Let ABCD be the parallelogram circumscribing the circle with centre O, such that AB, BC, CD and DA touch the circle at points P, Q, R and S respectively.

We know that the tangents drawn to a circle from an exterior point are equal in length.

$\therefore$ $AP = AS$ ...(1)

$BP = BQ$ ...(2)

$CR = CQ$ ...(3)

$DR = DS$ ...(4) 1

Adding (1), (2), (3) and (4), we get

$AP + BP + CR + DR = AS + BQ + CQ + DS$

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\therefore$ $AB + CD = AD + BC$ ...(5) 1

Also, $AB = DC$

and $AD = BC$(Opposite sides of parallelogram ABCD) ½

Putting in (5), we get, $2AB = 2AD$

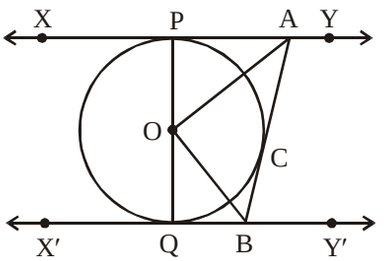
or $AB = AD.$

$\therefore$ $AB = BC = DC = AD.$

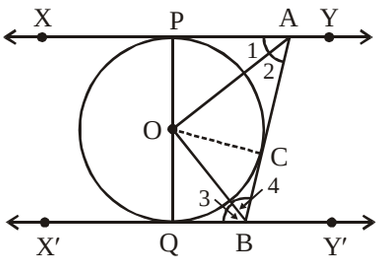
Since a parallelogram with equal adjacent sides is a rhombus, so ABCD is a rhombus. ½

Or

In the figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X′Y′ at B. What is the measure of ∠AOB ?



Sol.



Join OC.

In ΔOPA and ΔOCA,

$OP = OC$ (Radii of same circle)

$PA = CA$ (Lengths of two tangents from an external point) 1

$AO = AO$ (Common)

Therefore, $\Delta OPA \cong \Delta OCA$ (By SSS congruency criterion) ½

Hence, $\angle 1 = \angle 2$ (CPCT) ½

Similarly, $\angle 3 = \angle 4$

$\angle PAB + \angle QBA = 180°$ (Co-interior angles are supplementary as XY || X′Y′) ½

$2\angle 2 + 2\angle 4 = 180°$

$\angle 2 + \angle 4 = 90°$ ...(1) ½

$\angle 2 + \angle 4 + \angle AOB = 180°$ (Angle sum property)

Using (1), we get, $\angle AOB = 90°$.

31. Two coins are tossed simultaneously. What is the probability of getting
(*i*) At least one head?
(*ii*) At most one tail?
(*iii*) A head and a tail?

Sol. (*i*) P (At least one head) = $\frac{3}{4}$ 1

(*ii*) P(At most one tail) = $\frac{3}{4}$ 1

(*iii*) P(A head and a tail) = $\frac{2}{4} = \frac{1}{2}$ 1

SECTION – D

Section-D consists of 4 questions of 5 marks each.

32. To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool.

Sol. Let the time taken by larger pipe alone to fill the tank = x hours.
Therefore, the time taken by the smaller pipe = $x + 10$ hours. ½

Water filled by larger pipe running for 4 hours = $\frac{4}{x}$ of the pool.

Water filled by smaller pipe running for 9 hours = $\frac{9}{x+10}$ of the pool.

We know that

$$\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$$ 1

Which on simplification gives :

$$x^2 - 16x - 80 = 0$$ 1

$$x^2 - 20x + 4x - 80 = 0$$

$$x(x - 20) + 4(x - 20) = 0$$

$$(x + 4)(x - 20) = 0$$

$$x = -4,\ 20$$ 1

x cannot be negative.

Thus, $x = 20$ ½

$x + 10 = 30$ ½

Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours. ½

Or

In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr from its usual speed and the time of the flight increased by 30 min. Find the scheduled duration of the flight.

Sol. Let the usual speed of plane be x km/hr
and the reduced speed of the plane be $(x - 200)$ km/hr ½

Distance = 600 km [Given]

According to the question,
(Time taken at reduced speed) – (Schedule time) = 30 minutes = 0.5 hours.

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$$ 1

Which on simplification gives :

$x^2 - 200x - 240000 = 0$ 1

$x^2 - 600x + 400x - 240000 = 0$

$x(x - 600) + 400(x - 600) = 0$

$(x - 600)(x + 400) = 0$

$x = 600$ or $x = -400$ 1

But speed cannot be negative. ½

$\therefore$ The usual speed is 600 km/hr and so ½

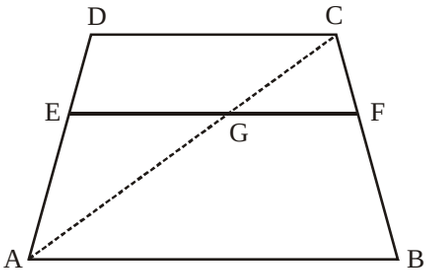
the scheduled duration of the flight is $\frac{600}{600} = 1$ hour. ½

33. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non-parallel sides in the same ratio.

Sol. For the theorem :

Given, To prove, Construction and figure 1½

Proof 1½



½

Let ABCD be a trapezium DC || AB and EF is a line parallel to AB and hence to DC.

To prove : $\frac{DE}{EA} = \frac{CF}{FB}$

Construction : Join AC, meeting EF in G.

Proof : In ΔABC, we have

$GF \parallel AB$

$\frac{CG}{GA} = \frac{CF}{FB}$ [By BPT] ...(1) ½

In ΔADC, we have

$EG \parallel DC$ (EF || AB and AB || DC) ½

$\frac{DE}{EA} = \frac{CF}{FB}$ [By BPT] ...(2)

From (1) & (2), we get,

$\frac{DE}{EA} = \frac{CF}{FB}$ ½

34. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per m^2, find the amount shared by each school to set up the tents.

Sol. Radius of the base of cylinder (r) = 2.8 m = Radius of the base of the cone (r)

Height of the cylinder (h) = 3.5 m

Height of the cone (H) = 2.1 m.

Slant height of conical part (l) $= \sqrt{r^2 + H^2}$

$= \sqrt{(2.8)^2 + (2.1)^2}$ 1

$= \sqrt{7.84 + 4.41}$ 1

$= \sqrt{12.25} = 3.5$ m

Area of canvas used to make one tent = CSA of cylinder + CSA of cone 1

$= 2 \times \pi \times 2.8 \times 3.5 + \pi \times 2.8 \times 3.5$

$= 61.6 + 30.8$

$= 92.4\ m^2$ 1

Cost of 1500 tents at ₹ 120 per sq.m $= 1500 \times 120 \times 92.4$ 1

$= 16{,}632{,}000$

Share of each school to set up the tents $= \frac{16632000}{50}$

$=$ ₹ 332,640

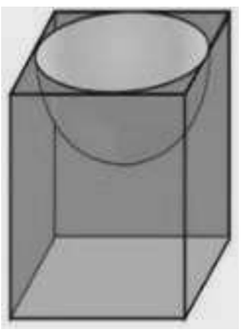
Or

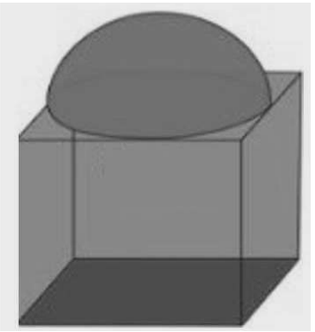
There are two identical solid cubical boxes of side 7 cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find

(*i*) the ratio of the total surface area of the two new solids formed

(*ii*) volume of each new solid formed.

Sol. First Solid Second Solid





(*i*) SA for first new solid (S_1) :

$6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2 = 294 + 77 - 38.5$ 1

$= 332.5\ cm^2$

SA for second new solid (S_2) :

$6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2 = 294 + 77 - 38.5$ 1

$= 332.5\ cm^2$ 1

So, $S_1 : S_2 = 1:1$

(*ii*) Volume for first new solid (V_1) $= 7 \times 7 \times 7 - \frac{2}{3}\pi \times 3.5^2$

$= 343 - \frac{539}{6} = \frac{1519}{6}\ cm^3$ 1

Volume for second new solid (V_2) $= 7 \times 7 \times 7 + \frac{2}{3}\pi \times 3.5^2$

$= 343 + \frac{539}{6} = \frac{2597}{6}\ cm^3$ 1

35. The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class interval	Frequency
0–100	2
100–200	5
200–300	x
300– 400	12
400–500	17
500–600	20
600–700	y
700–800	9
800–900	7
900–1000	4

Sol. Median = 525, so Median Class = 500 – 600 ½

Class interval	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$

1½

$$76 + x + y = 100 \Rightarrow x + y = 24 \quad ...(i)$$ 1

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$ ½

Since, $l = 500$, $h = 100$, $f = 20$, $cf = 36 + x$ and $n = 100$
Therefore, putting the value in the Median formula, we get;

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$ ½

so, $x = 9$

$y = 24 - x$ [From eq. (i)]

$y = 24 - 9 = 15$

Therefore, the value of $x = 9$ ½
and $y = 15$. ½

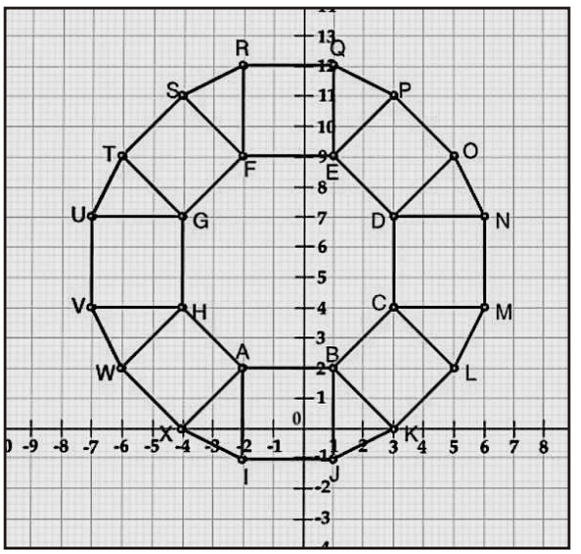
SECTION – E

Case study based questions are compulsory.

36. A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find

tessellation patterns on floors, walls, paintings etc. Shown below is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons.

A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern.



Use the above figure to answer the questions that follow :

(*i*) What is the length of the line segment joining points B and F?

(*ii*) The centre 'Z' of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then, what are the coordinates of Z?

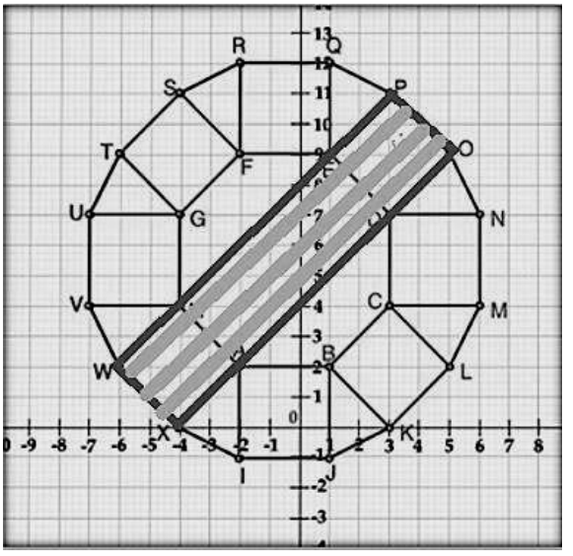
(*iii*) What are the coordinates of the point on *y*-axis equidistant from A and G?

Sol. (*i*) B(1, 2), F(– 2, 9)

$$BF^2 = (-2 - 1)^2 + (9 - 2)^2$$
$$= (-3)^2 + (7)^2$$
$$= 9 + 49 = 58$$

So, $BF = \sqrt{58}$ units 1

(*ii*)



W(–6, 2), X(– 4,0), O(5, 9), P(3, 11) ½

Clearly WXOP is a rectangle.

Point of intersection of diagonals of a rectangle is the mid-point of each of the the diagonals. So the required point is mid-point of WO or XP

$$= \left(\frac{-6+5}{2}, \frac{2+9}{2}\right) = \left(\frac{-1}{2}, \frac{11}{2}\right)$$

or $\left(\frac{-4+3}{2}, \frac{0+11}{2}\right) = \left(\frac{-1}{2}, \frac{11}{2}\right)$ ½

(*iii*) A(–2, 2), G(–4, 7)

Let the point on y-axis be Z(0, y) ½

$$AZ^2 = GZ^2 \quad ½$$

$$(0+2)^2 + (y-2)^2 = (0+4)^2 + (y-7)^2$$

$$(2)^2 + y^2 + 4 - 4y = (4)^2 + y^2 + 49 - 14y$$

$$8 - 4y = 65 - 14y$$

$$10y = 57$$

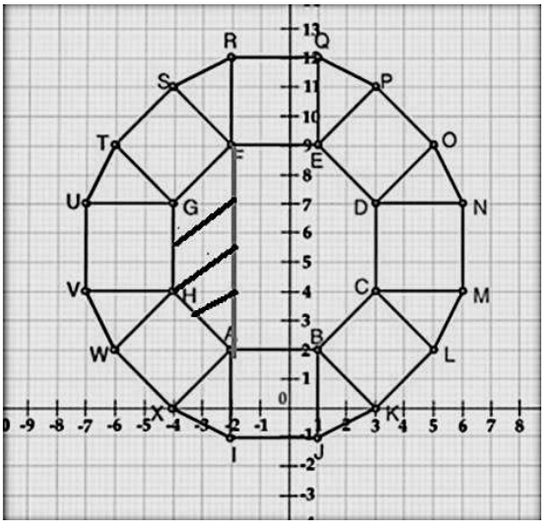
So, $y = 5.7$ ½

i.e., the required point is (0, 5.7) ½

Or

What is the area of Trapezium AFGH?

Sol.



A(–2, 2), F(–2, 9), G(–4, 7), H(–4, 4)

Clearly, GH = 7 – 4 = 3 units ½

AF = 9 – 2 = 7 units ½

So, height of the trapezium AFGH = 2 units

So, area of AFGH $= \frac{1}{2}(AF + GH) \times$ Height

$= \frac{1}{2}(7 + 3) \times 2$ ½

= 10 sq. units ½

37. The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

(*i*) If the first circular row has 30 seats, how many seats will be there in the 10th row?

(*ii*) For 1500 seats in the auditorium, how many rows need to be there?

Sol. (*i*) Since each row is increasing by 10 seats, so it is an AP with first term $a = 30$, and common difference $d = 10$. ½

So, number of seats in 10th row $= a_{10} = a + 9d$

$= 30 + 9 \times 10 = 120$ ½

(*ii*) $S_n = \frac{n}{2}[2a + (n - 1)\, d]$ ½

$1500 = \frac{n}{2}[2 \times 30 + (n - 1)\, 10]$

$3000 = 50n + 10n^2$

$n^2 + 5n - 300 = 0$ ½

$n^2 + 20n - 15n - 300 = 0$

$(n + 20)(n - 15) = 0$ ½

Rejecting the negative value, $n = 15$. ½

Or

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?

(*iii*) If there were 17 rows in the auditorium, how many seats will be there in the middle row?

Sol. No. of seats already put up to the 10th row = S_{10} ½

$S_{10} = \frac{10}{2}\{2 \times 30 + (10 - 1)\, 10)\}$ ½

$= 5(60 + 90) = 750$ ½

So, the number of seats still required to be put are 1500 – 750 = 750 ½

(*iii*) If number of rows = 17,

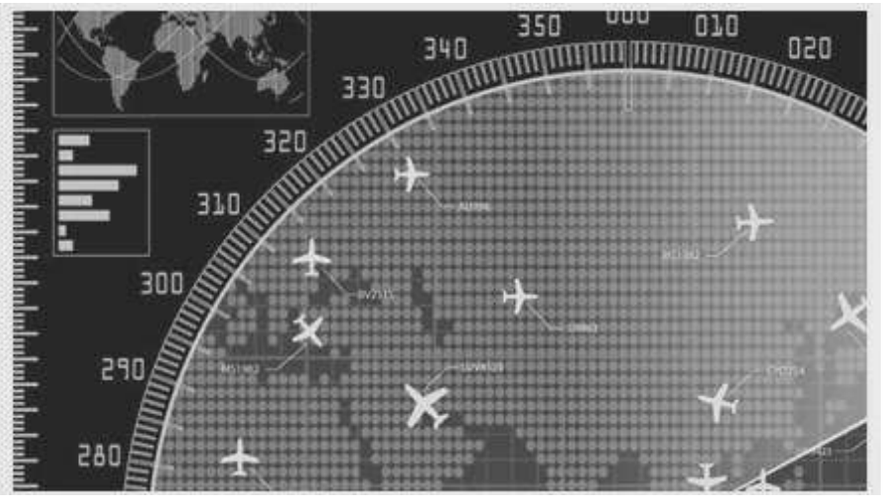
then the middle row is the 9th row ½

$a_9 = a + 8d$

$= 30 + 80$

$= 110$ seats ½

38. We all have seen the airplanes flying in the sky but might have not thought of how they actually reach the correct destination. Air Traffic Control (ATC) is a service provided by ground-based air traffic controllers who direct aircraft on the ground and through a given section of controlled airspace, and can provide advisory services to aircraft in non-controlled airspace. Actually, all this air traffic is managed and regulated by using various concepts based on coordinate geometry and trigonometry.

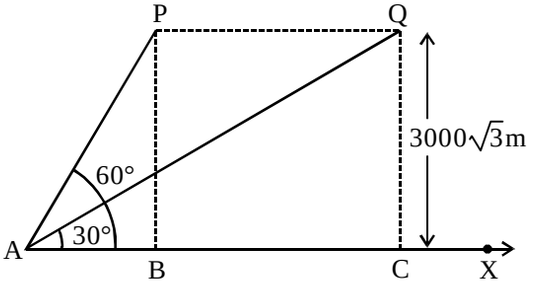


At a given instance, ATC finds that the angle of elevation of an airplane from a point on the ground is 60°. After a flight of 30 seconds, it is observed that the angle of elevation changes to 30°. The height of the plane remains constantly as $3000\sqrt{3}$ m. Use the above information to answer the questions that follow :

(*i*) Draw a neat labelled figure to show the above situation diagrammatically.

(*ii*) What is the distance travelled by the plane in 30 seconds?

Sol. (*i*) Diagram is as shown below.



1

P and Q are the two positions of the plane flying at a height of $3000\sqrt{3}$ m.

A is the point of observation.

(*ii*) From ΔPAB, $\tan 60° = \frac{PB}{AB}$ or $\sqrt{3} = \frac{3000\sqrt{3}}{AB}$ 1

So, $AB = 3000$ m

From ΔQAC, $\tan 30° = \frac{QC}{AC}$ ½

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{AC}$$

$AC = 9000$ m ½

Distance covered $= 9000 - 3000$

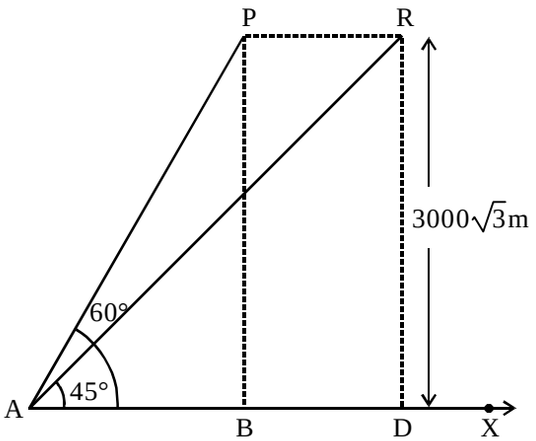
$= 6000$ m. ½

Or

Keeping the height constant, during the above flight, it was observed that after $15(\sqrt{3}-1)$ seconds, the angle of elevation changed to 45°. How much is the distance travelled in that duration.

(*iii*) What is the speed of the plane in km/hr.

Sol.



½

(*ii*) From ΔPAB, $\tan 60° = \frac{PB}{AB}$

or $\sqrt{3} = \frac{3000\sqrt{3}}{AB}$ ½

So, $AB = 3000$ m

From ΔRAD, $\tan 45° = \frac{RD}{AD}$ ½

$1 = \frac{3000\sqrt{3}}{AD}$ ½

$AD = 3000\sqrt{3}$ m ½

Distance covered $= 3000\sqrt{3}$ m $- 3000$ m

$= 3000(\sqrt{3}-1)$ m. ½

(*iii*) $\text{Speed} = \frac{6000}{30}$ ½

$= 200$ m/s

$= 200 \times \frac{3600}{1000}$ ½

$= 720$ km/hr

Alternatively : $\text{Speed} = \frac{3000\,(\sqrt{3}-1)}{15\,(\sqrt{3}-1)}$ ½

$= 200$ m/s

$= 200 \times \frac{3600}{1000}$ ½

$= 720$ km/hr

2 SAMPLE QUESTION PAPER

MATHEMATICS (Standard)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 5 marks, 2 Questions of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. If a and b are two positive numbers, H and L are their HCF and LCM respectively, then :

(A) $H=\frac{L}{a\times b}$ (B) $a=\frac{b\times L}{H}$ (C) $a=\frac{b\times H}{L}$ (D) $a \times b = H \times L$

2. HCF of two prime numbers is :

(A) 0 (B) 1 (C) 2 (D) 3

3. The equation of $x(x - 1) + 9 = 2x(x^3 + 9)$ is a

(A) linear equation (B) quadratic equation

(C) biquadratic equation (D) cubic equation

4. The cost of production per unit for two products, A and B, are ₹ 100 and ₹ 80 respectively. In a week, the total production cost is ₹ 32000. In the next week, the production cost reduces by 20%, and the total cost of producing the same number of units of each product is ₹ 25600. Which of these are the equations that can be used to find the number of units of A, x, and the number of units of B, y?

(A) $100x + 80y = 25600$ and $80x + 64y = 32000$

(B) $100x + 64y = 32000$ and $80x + 100y = 25600$

(C) $80x + 80y = 32000$ and $100x + 64y = 25600$

(D) $100x + 80y = 32000$ and $80x + 64y = 25600$

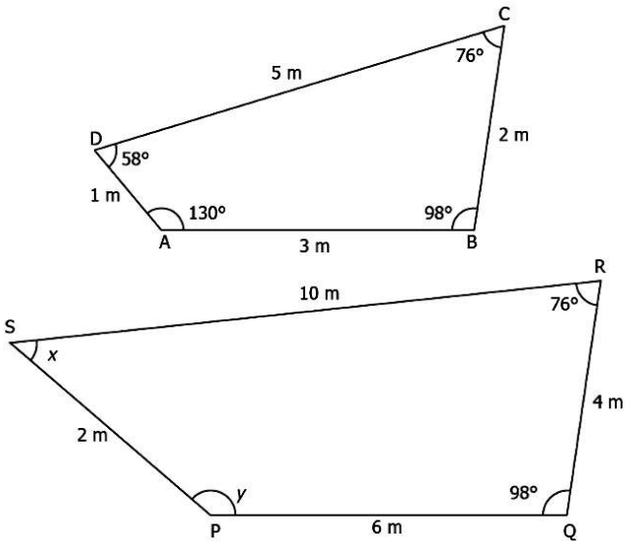
5. A teacher asked students to find the roots of the equation $\frac{x}{x+1}+\frac{x+1}{x}-\frac{34}{15}=0$. Two students, Ravi and Ankit gave following answers. Ravi said one of the roots is $\frac{3}{2}$. Ankit said one of the roots is $\frac{(-5)}{2}$.

Who is correct?

(A) Ravi (B) Ankit

(C) Both Ravi and Ankit (D) Neither Ravi nor Ankit

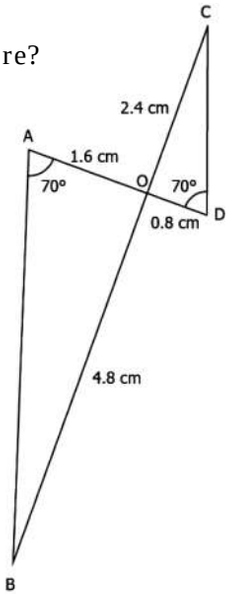
6. Two similar figures are shown.



What are the values of x and y?

(A) $x = 58°$, $y = 130°$ (B) $x = 98°$, $y = 76°$ (C) $x = 82°$, $y = 84°$ (D) $x = 130°$, $y = 84°$

7. Consider the figure given alongside.



Which of the following statement is correct about the triangles in the figure?

(A) $\Delta AOB \sim \Delta DOC$ because

$$\frac{AO}{DO} = \frac{BO}{CO}.$$

(B) $\Delta AOB \sim \Delta DOC$ because

$$\angle AOB = \angle DOC.$$

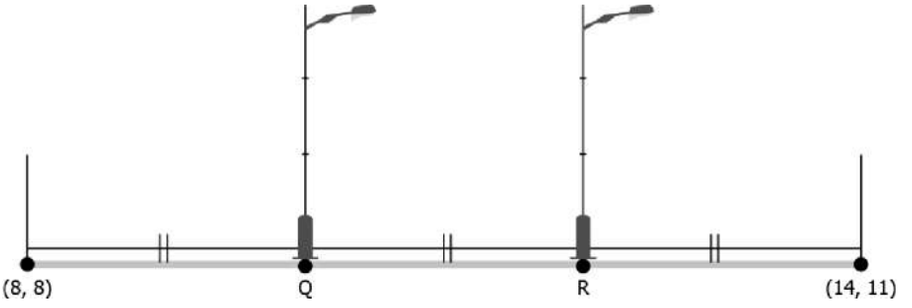
(C) $\Delta AOB \sim \Delta DOC$ because

$$\frac{AO}{DO} = \frac{BO}{CO} \text{ and}$$
$$\angle AOB = \angle DOC.$$

(D) $\Delta AOB \sim \Delta DOC$ because

$$\frac{AO}{DO} = \frac{BO}{CO} \text{ and}$$
$$\angle BAO = \angle CDO.$$

8. Two poles are to be installed on an elevated road as shown in the diagram. The diagram also shows the starting and ending points of the road.

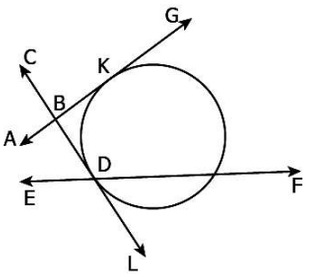


Which of the following are the coordinates of the foot of the poles?

(A) Q (10, 9) and R(12, 8) (B) Q (10, 8) and R (12, 11)

(C) Q (10, 9) and R(12, 10) (D) Q (–10, 9) and R(0, 11)

9. A figure is shown below.



Which of the following is true?

(A) Lines AG and CL are the tangents and line EF is a secant to the circle.

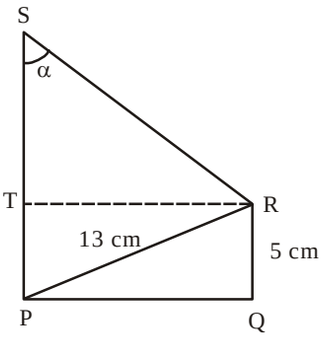
(B) Lines AG and CL are the secants and line EF is a tangent to the circle.

(C) Line AG is a tangent and lines EF and CL are the secants to the circle.

(D) Line AG is a secant and lines EF and CL are the tangents to the circle.

10. In the given figure, if PS = 14 cm, the value of tan a is equal to :

(A) $\frac{4}{3}$ (B) $\frac{14}{3}$ (C) $\frac{5}{3}$ (D) $\frac{13}{3}$



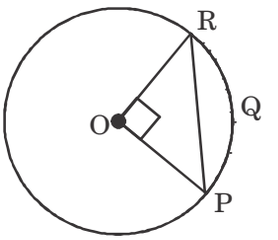
11. In ΔABC right angled at B, AC = 13 cm, AB = 5 cm then sin A is equal to

(A) $\frac{5}{13}$ (B) $\frac{5}{12}$ (C) $\frac{12}{13}$ (D) $\frac{13}{12}$

12. If sec A = 2, where A is acute, then tan A = ?

(A) 2 (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) 1

13. Observe the figure given alongside.

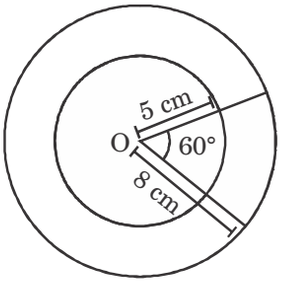


What is the area of the segment PQR, if the radius of the circle is 7 cm ? $\left(\text{Use } \pi = \frac{22}{7}\right)$

(A) 11 cm^2 (B) 14 cm^2

(C) 17.3 cm^2 (D) 91 cm^2

14. Two concentric circles of radius 8 cm and 5 cm are shown below, and a sector forms an angle of 60° at the centre O. What is the area of the shaded region?



(A) 38π cm^2 (B) $\frac{77\pi}{2}$ cm^2

(C) $\frac{11\pi}{2}$ cm^2 (D) $\frac{195\pi}{2}$ cm^2

15. If the radius *r* of the base of a right circular cylinder is halved, keeping the height *h* same, the surface area of the reduced cylinder is

(A) $\pi rh + \pi r^2$ (B) $2\pi rh + \pi r^2$ (C) $\pi rh + \frac{\pi r^2}{2}$ (D) $2\pi r^2 + \pi rh$

16. In the formula, mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$, f_2 is

(A) frequency of the modal class
(B) frequency of the second class
(C) frequency of the class preceding the modal class
(D) frequency of the class succeeding the modal class

17. Monthly pocket money of 50 students of a class are given in the following distribution :

Monthly pocket money (in ₹)	0–50	50–100	100–150	150–200	200–250	250–300
No. of students	2	7	8	30	12	1

The class mark of the modal class is :

(A) 125 (B) 150 (C) 175 (D) 225

18. When four coins are tossed simultaneously, which of the following represents the sample space?

(A)

HHHH	HHHT	HHTH	HTHH
THHH	HHTT	TTHH	HTTT
THTT	TTHT	TTTH	TTTT

(B)

HHHH	HHHT	HHTH	HTHH
THHH	THHT	HTTH	HTTT
THTT	TTHT	TTTH	TTTT

(C)

HHHH	HHHT	HHTH	HTHH
THHH	HHTT	HTHT	HTTH
THHT	THTH	TTHH	HTTT
THTT	TTHT	TTTH	TTTT

(D)

HHHH	HHHT	HHTH	HTHH
THHH	HHTT	HTHT	HTTH
HTTH	HTHT	TTHH	HTTT
THTT	TTHT	TTTH	TTTT

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true but (R) is false.
(D) (R) is true but (A) is false.

19. **(A) :** HCF of 95 and 152 is 19.
(R) : LCM of 95 and 152 is 95 × 152.

20. **(A) :** Mid-point of the line-segment joining the points (2, –3) and (3, 3) is $\left(\frac{5}{2}, \frac{1}{2}\right)$.

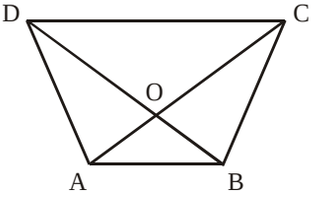
(R) : Mid-point of line segment joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

SECTION – B

Section-B consists of 5 questions of 2 marks each.

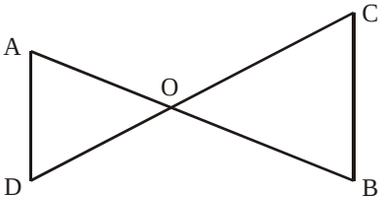
21. A father is three times as old as his son. After five years, his age will be two and a half times as old as his son. Represent this situation algebraically only.

22. In a trapezium, diagonals AC and BD meet at O (see figure). Prove that $\Delta AOB \sim \Delta COD$.

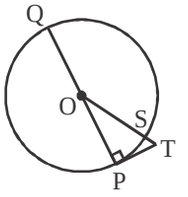


Or

In the given figure, if OA.OB = OC.OD, show that ∠A = ∠C and ∠B = ∠D.

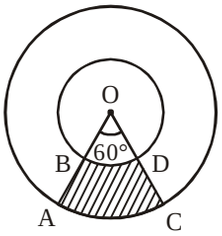


23. In the figure, PQ is a diameter and PT is a tangent of the circle with centre O. If ∠QOS = 150°, find ∠PTS.



24. If $\tan A = \frac{5}{12}$, find the value of (sin A + cos A) × sec A.

25. In the given figure, the radii of two concentric circles with centre O are 7 cm and 14 cm and ∠AOC = 60°. Find that area of the shaded region.



Or

With the vertices A, B and C of an equilateral triangle ABC as centres, arcs are drawn with radii 7 cm each. If the side of the triangle is 40 cm, then find the area of the region of the triangle not included in the sectors. (Use π = 3.14)

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Find HCF of 378, 180 and 420 by prime factorisation method. Is HCF × LCM of three numbers equal to the product of the three numbers ?

27. If α and β are zeroes of a quadratic polynomial $x^2 - x - 30$, then form a quadratic polynomial whose zeroes are 2 – α and 2 – β.

28. For what values of K will the following pair of linear equation have infinitely many solutions ?

$$Kx + 3y - (K - 3) = 0$$
$$12x + Ky - K = 0$$

Or

If sum of two positive numbers is 108 and the difference of these numbers is 8, then find the numbers.

29. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other.

30. If $\sec\theta = \frac{13}{5}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} + 3\cos\theta$.

Or

If 4 tan θ = 3, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$.

31. A month is selected at random in a year. Find the probability that the month is of 31 days.

SECTION – D

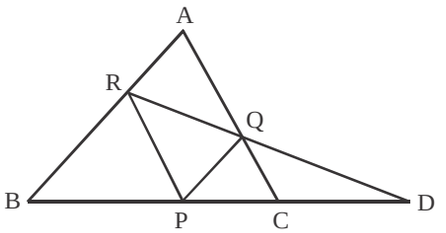
Section-D consists of 4 questions of 5 marks each.

32. A train travels a distance of 300 km at a constant speed. If speed of the train is increased by 5 km/h, the journey will take 2 hours less. Find the speed of the train.

Or

The sum of the ages of a boy and his brother is 25 years and the product of their ages (in years) is 126. Find their present ages.

33. In the given figure, PQ || BA and PR || CA. If PD = 12 cm, find BD × CD.



34. A medicine capsule is in the form of a cylinder with two hemispherical ends. The radius of the capsule is 3.5 mm and length of the capsule is 12 mm. Find its total surface area. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Or

A tent is in the shape of a cylinder surmounted by a conial top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2.

35. Cost of living index for some period is given in the following frequency distribution :

Index	No. of weeks
1500 – 1600	3
1600 – 1700	11
1700 – 1800	12
1800 – 1900	7
1900 – 2000	9
2000 – 2100	8
2100 – 2200	2

Find the mode and median for above data.

SECTION – E

Case study based questions are compulsory.

36. Saving money is a good habit and it should be inculcated in children from the beginning. A father brought a piggy bank for his son Aditya. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(*i*) How many five rupee coins will be put by Aditya on the 5th day ?

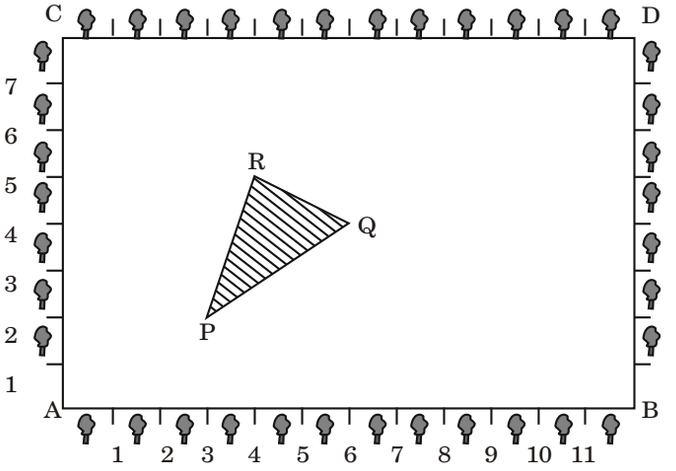
(*ii*) How many five rupee coins will be put by Aditya on the 10th day ?

(*iii*) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it.

Or

Find the total money he saved if he put coins for 19 days.

37. Students of Class X were allotted a rectangular plot ABCD for garding activities. Saplings of Gulmohar were planted on the boundary such that distance between any two consecutive saplings is 1 m. Further, there is triangular grassy lawn PQR in the plot as shown in the figure. On the remaing part, students are to sow seeds of flowering plants.



(*i*) Taking A is origin, write the coordinates of the point P.

(*ii*) Taking A is origin, write the coordinates of the point Q.

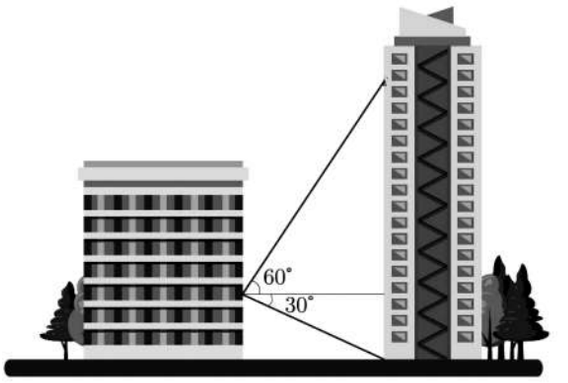
(*iii*) Find the coordinates of the mid-point of the line-segment PQ.

Or

If B is taken as origin, then what will be the coordinates of the mid-point of QR ?

38. From his hotel room window on the fourth floor, Ranjan notices some window washers high above him on the hotel across the street.

Curious as to their height above ground, he quickly estimates the buildings are 60 m apart, the angle of elevation to the washers is about 60°, and the angle of depression to the base of the hotel is about 30°.



(*i*) Draw a well-labelled figure for the above information.

(*ii*) How high above ground is the window of Ranjan's hotel room?

(*iii*) How high above ground are the washers?

Or

Find the distance between Ranjan and washers.

Answers

1. (D) $a \times b = H \times L$ **2.** (B) 1 **3.** (D) cubic equation

4. (D) $100x + 80y = 32000$ and $80x + 64y = 25600$

5. (C) Both Ravi and Ankit **6.** (A) $x = 58°$, $y = 130°$

7. (C) $\Delta AOB \sim \Delta DOC$ because $\frac{AO}{DO} = \frac{BO}{CO}$ and $\angle AOB = \angle DOC$.

8. (C) Q (10, 9) and R(12, 10)

9. (A) Lines AG and CL are the tangents and line EF is a secant to the circle.

10. (A) $\frac{4}{3}$ **11.** (C) $\frac{12}{13}$ **12.** (C) $\sqrt{3}$ **13.** (B) 14 cm^2

14. (D) $\frac{195\pi}{2} cm^2$ **15.** (C) $\pi rh + \frac{\pi r^2}{2}$

16. (D) frequency of the class succeeding the modal class

17. (C) 175 **18.** (C) **19.** (C) (A) is true but (R) is false.

20. (D) (R) is true but (A) is false.

21. Let the present ages of son and father be x and y years, respectively.

Then, $y = 3x$

or $3x - y = 0$ ...(1)

Five years later, father's age = $y + 5$ and son's age = $x + 5$.

According to question,

$$y + 5 = 2\frac{1}{2}(x + 5)$$

or $$y + 5 = \frac{5}{2}(x + 5)$$

or $2y + 10 = 5x + 25$

or $5x - 2y + 15 = 0$ ...(2)

22. $AB \parallel CD$

So, $\angle ABO = \angle CDO$ (Alternate interior angles)

and $\angle AOB = \angle COD$ (Vertically opposite angles)

Som $\Delta AOB \sim \Delta COD$ (By AA criterion), **proved.**

Or

We are given :

$OA.OB = OC.OD$

$\Rightarrow \frac{OA}{OD} = \frac{OC}{OB}$ ...(1)

Also, $\angle AOD = \angle COB$ ...(2)

(Vertically opposite angles)

So, from (1) and (2),

$\Delta AOD \sim \Delta COB$ (By SAS similarity criterion)

So, $\angle A = \angle C$ and $\angle B = \angle D$, proved.

23. We have :

$\angle OPT = 90°$ (Angle between tangent and radius) ...(1)

Also, $\angle POT = 180° - \angle QOS$ (Linear pair)

$= 180° - 150° = 30°$ ...(2)

Now, in ΔPOT, we have :

$\angle PTO + \angle POT + \angle OPT = 180°$ (Angle sum property of a triangle)

$\Rightarrow$ $\angle PTO + 30° + 90° = 180°$ [From (1) and (2)]

$\Rightarrow$ $\angle PTO = 180° - 30° - 90° = 60°$.

So, $\angle PTS = \angle PTO = 60°$.

24. $\tan A = \frac{5}{12}$

Let BC = $5k$ and AC = $12k$

By Pythagoras Theorem,

$$AB^2 = AC^2 + BC^2$$
$$= (12k)^2 + (5k)^2$$
$$= 144k^2 + 25k^2$$
$$AB^2 = 169k^2$$
$$\Rightarrow \quad AB = 13k$$

$$\sin A = \frac{BC}{AB} = \frac{5k}{13x} = \frac{5}{13}$$

$$\cos A = \frac{AC}{AB} = \frac{12k}{13k} = \frac{12}{13}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{12/13} = \frac{13}{12}$$

$$(\sin A + \cos A)\sec A = \left(\frac{5}{13} + \frac{12}{13}\right)\frac{13}{12} = \frac{17}{13} \times \frac{13}{12} = \frac{17}{12}$$

25. We have, r = 7 cm, R = 14 cm and $\theta = 60°$.

So, area of the shaded region $= \frac{\theta}{360} \times \pi R^2 - \frac{\theta}{360} \times \pi r^2$.

$$= \frac{60}{360}\pi(R^2 - r^2).$$

$$= \frac{1}{6} \times \frac{22}{7}[(14)^2 - (7)^2]$$

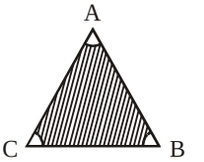
$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 7 = 11 \times 7$$

$= 77$ cm^2.

Or

Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (40)^2$

$= 400\sqrt{3}$ cm^2.



Sum of the areas of the three sectors $= \frac{(60° + 60° + 60°)}{360°} \times \pi \times (7)^2$ cm^2.

$= \frac{1}{2} \times 3.14 \times 49$ cm^2.

$= 1.57 \times 49$ cm^2

$= 76.93$ cm^2.

Hence, area of the triangle not included in the sectors = $(400\sqrt{3} - 76.93)$ cm^2.

26. Prime factorisation of 378, 180 and 420 are:

$378 = 2 \times 3^3 \times 7$

$180 = 2^2 \times 3^2 \times 5$

$420 = 2^2 \times 3 \times 5 \times 7$

So, HCF $= 2 \times 3 = 6$.

No, HCF × LCM of three numbers is not equal to the product of the three numbers by actual multiplication.

27. α and β are zeroes of the quadratic polynomial $x^2 - x - 30$.

$$\therefore \quad \alpha + \beta = \frac{-(-1)}{1} = 1 \qquad ...(1)$$

and $$\alpha\beta = \frac{-30}{1} = -30. \qquad ...(2)$$

Now, sum of $(2 - \alpha)$ and $(2 - \beta) = 2 - \alpha + 2 - \beta$

$= 4 - (\alpha + \beta)$

$= 4 - 1 = 3$ [From (1)]

and product of $(2 - \alpha)$ and $(2 - \beta) = (2 - \alpha)(2 - \beta)$

$= 4 - 2\alpha - 2\beta + \alpha\beta$

$= 4 - 2(\alpha + \beta) + \alpha\beta$

$= 4 - 2 \times 1 - 30 = -28.$ [From (1) and (2)]

Hence, required polynomial is $= x^2 -$ (Sum of zeroes) $x +$ (Product of zeroes)

$= x^2 - 3x + (-28) = x^2 - 3x - 28.$

28. We have :

$$Kx + 3y - (K - 3) = 0$$
$$12x + Ky - K = 0$$

For infinitely many solutions,

$$\frac{K}{12} = \frac{3}{K} = \frac{K-3}{K}$$

$$\Rightarrow \quad \frac{K}{12} = \frac{3}{K}$$

or $$K^2 = 12 \times 3 = 36$$

or $$K = \pm 6$$

For K = 6, $$\frac{K}{12} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{3}{K} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{K-3}{2} = \frac{6-3}{6} = \frac{3}{6} = \frac{1}{2}$$

i.e., $$\frac{K}{12} = \frac{3}{K} = \frac{K-3}{K} = \frac{1}{2} \text{ for } K = 6$$

Hence, the given pair of linear equations will have infinitely many solutions for K = 6.

For K = – 6, the above conditions are not satisfied.

So, K = – 6 is rejected.

Or

Let required positive numbers be x and y. Then, by Ist condition,

$$x + y = 108 \qquad ...(i)$$

and by IInd condition,

$$x - y = 8 \qquad ...(ii) \quad [\text{when } x > y]$$

or $$y - x = 8 \qquad ...(iii) \quad [\text{when } y > x]$$

Adding (i) and (ii), we get

$$2x = 116 \quad \Rightarrow \quad x = 58.$$

Putting $x = 58$ in (i), we get

$$y = 108 - 58 \Rightarrow y = 50.$$

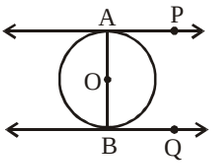
Similarly, solving (i) and (iii), we get $y = 58$ and $x = 50$.

if $x = 58 \Rightarrow y = 50$

and if $x = 50 \Rightarrow y = 58$

So, required numbers are 58 and 50.

29. **Given :** Diameter AB of a circle with centre O. Also, AP and BQ are tangents to the circle.
To prove : AP || BQ.



Proof : $\angle OAP = 90°$ (Angle between radius and tangent) ...(1)

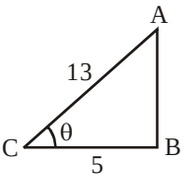
Similarly, $\angle OBQ = 90°$ (Angle between radius and tangent) ...(2)

So, $\angle OAP + \angle OBQ = 90° + 90° = 180°$ [From (1) and (2)]

But these are interior angles on the same side of the transversal AB to lines AP and BQ.
Hence, AP || BQ.

30. ***Hint :*** Pythagoras theorem will be used to find other trigonometric ratios with the help of $\sec\theta = \frac{13}{5}$.

We have : $\sec\theta = \frac{13}{5}$.



$\Rightarrow \quad \frac{1}{\cos\theta} = \frac{13}{5}$

$\Rightarrow \quad \cos\theta = \frac{5}{13}$.

Therefore, if in ΔABC, BC = 5 and AC = 13,

then $AB = \sqrt{169 - 25} = \sqrt{144} = 12.$

So, $\sin\theta = \frac{12}{13}$.

Therefore, $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} + 3\cos\theta = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}} + 3\times\frac{5}{13} = \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} + \frac{15}{13}$

$$= \frac{\frac{24-15}{13}}{\frac{48-45}{13}} + \frac{15}{13} = \frac{\frac{9}{13}}{\frac{3}{13}} + \frac{15}{13}.$$

$$= \frac{9}{13}\times\frac{13}{3} + \frac{15}{13}$$

$$= 3 + \frac{15}{13} = \frac{39+15}{13}.$$

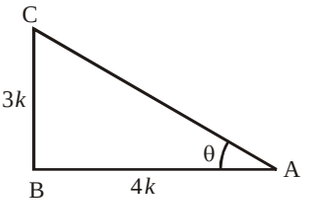
$$= \frac{54}{13} = 4\frac{2}{13}.$$

Or

We have :

$$\tan\theta = 3$$

$$\Rightarrow \quad \tan\theta = \frac{3}{4}.$$



Let us take a rt. ΔABC right angled at B,
if $AB = 4k$ and $BC = 3k$, then

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{(4k)^2 + (3k)^2}$$

$$= \sqrt{16k^2 + 9k^2} = \sqrt{25k^2} = 5k.$$

So, $\sin\theta = \frac{3k}{5k} = \frac{3}{5}$

and $\cos\theta = \frac{4k}{5k} = \frac{4}{5}$.

Therefore, $$\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1} = \frac{\frac{12}{5} - \frac{4}{5} + 1}{\frac{12}{5} + \frac{4}{5} - 1}$$

$$= \frac{\frac{12 - 4 + 5}{5}}{\frac{12 + 4 - 5}{5}} = \frac{\frac{13}{5}}{\frac{11}{5}} = \frac{13}{5} \times \frac{5}{11} = \frac{13}{11}.$$

31. Total number of months = 12.
Number of months with 31 days = 7
(Jan, March, May, July, August, October, December).

So, required probability $= \frac{7}{12}$.

[***Check :*** May write the answer as $\frac{6}{12}$.]

32. Let speed of the train be x km/hour.
As per given conditions,

$$\frac{300}{x} - \frac{300}{x + 5} = 2.$$

$$\Rightarrow \quad 300\frac{(x + 5 - x)}{x(x + 5)} = 2$$

$$\Rightarrow \quad 300 \times 5 = 2x^2 + 10x.$$

$$\Rightarrow \quad 2x^2 + 10x - 1500 = 0.$$

$$\Rightarrow \quad x^2 + 5x - 750 = 0$$

$$\Rightarrow \quad x^2 + 30x - 25x - 750 = 0$$

$$\Rightarrow \quad x(x + 30) - 25(x + 30) = 0$$

$$\Rightarrow \quad (x + 30)(x - 25) = 0.$$

$$\Rightarrow \quad x = -30 \quad \text{or} \quad x = 25.$$

Neglecting $x = -30$, the speed of the train is 25 km/hour.

Probable Error : Sometimes $x^2 + 5x - 750$ is factorised as $(x + 25)(x - 30)$ and answer obtained is 30 km/h.

Or

Let the ages of the two brothers be x and $25 - x$. (As sum is 25)

So, we get $x(25 - x) = 126$.

$\Rightarrow$ $25x - x^2 = 126$.

$\Rightarrow$ $x^2 - 25x + 126 = 0$

$\Rightarrow$ $x^2 - 18x - 7x + 126 = 0$

$\Rightarrow$ $x(x - 18) - 7(x - 18) = 0$

$\Rightarrow$ $(x - 18)(x - 7) = 0$

$\Rightarrow$ $x = 18$ or $x = 7$.

So, if age of one brother is 18, then the age of other brother is $25 - 18 = 7$.

Also, if age of one brother is 7, then the age of other brother = $25 - 7 = 18$.

Thus, ages of brothers are 18 years and 7 years.

33. In ΔDBR, we get $\frac{PD}{BD} = \frac{DQ}{DR}$ (Because PQ || BA, corrollary of BPT) ...(1)

In ΔDPR, we get $\frac{CD}{PD} = \frac{DQ}{DR}$ (Because QC || RP, corrollary of BPT) ...(2)

From (1) and (2), we get :

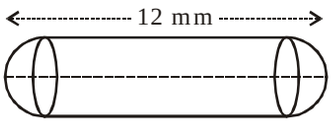
$$\frac{PD}{BD} = \frac{CD}{PD}$$

$\Rightarrow$ $PD^2 = BD \times CD$

$\Rightarrow$ $BD \times CD = (12)^2 = 12 \times 12$

$= 144 \text{ cm}^2$. [$\because$ PD = 12 cm]

34.



Height of the cylinder = (12 – 3.5 – 3.5) mm = 5 mm.

So, total surface area of the capsule = Curved surface area of cylinder + 2 × Curved surface area of hemisphere.

$= 2\pi rh + 2 \times 2\pi r^2$.

$= \left(2 \times \frac{22}{7} \times 3.5 \times 5 + 2 \times 2 \times \frac{22}{7} \times 3.5 \times 3.5\right) \text{mm}^2$.

$= \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 5 + 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{mm}^2$

$= (110 + 154) \text{ mm}^2$.

$= 264 \text{ mm}^2$.

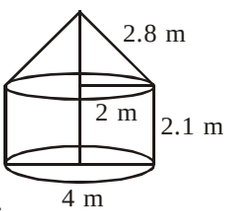
Or

Hint : Because for the base of the tent, no canvas is required, so canvas required will be curved surface are of cylinder + curved surface area of cone. After this cost can be calculated.

Diameter of cylinder = 4 m.

$\therefore$ Its radius = $\frac{4}{2}$ m = 2 m.

Hence, radius of cone = 2 m.



Hence, canvas required for tent = $2\pi rh + \pi rl$.

$= (2\pi \times 2 \times 2.1 + \pi \times 2 \times 2.8) \text{ m}^2$.

$= \pi \times (8.4 + 5.6) \text{ m}^2$

$= \frac{22}{7} \times 14 \text{ m}^2 = 44 \text{ m}^2$.

Its cost = ₹ 44 × 500 = ₹ 22000.

35. We have :

Index	Frequency	*c.f.*
1500 – 1600	3	3
1600 – 1700	11	14
1700 – 1800	12	26
1800 – 1900	7	33
1900 – 2000	9	42
2000 – 2100	8	50
2100 – 2200	2	52
Total	52	

For mode : Modal class is 1700 – 1800.

Here, $f_1 = 12$, $f_0 = 11$, $f_2 = 7$, $l = 1700$ and $h = 100$.

Hence, $$\text{mode} = 1700 + \left(\frac{12 - 11}{2 \times 12 - 11 - 7}\right) \times 100$$

$$= 1700 + \frac{1 \times 100}{24 - 18} = 1700 + \frac{100}{6}$$

$$= 1700 + \frac{50}{3} = 1700 + 16.66$$

$$= 1716.66.$$

For median : Median class is 1700 – 1800, since $\frac{N}{2} = \frac{52}{2} = 26$.

Here, N = 52, $l = 1700$, $cf = 14$, $f = 12$ and $h = 100$.

Hence, $$\text{median} = 1700 + \left(\frac{26 - 14}{12}\right) \times 100$$

$$= 1700 + \frac{12}{12} \times 100$$

$$= 1700 + 100 = 1800.$$

36. (*i*) If forms an AP 1, 2, 3, ...

So, on 5th day, he will put 1 + (5 – 1) = 5 coins.

(*ii*) On the 10th day, he will put 1 + (10 – 1) × 1 = 10 coins.

(*iii*) Using $S = \frac{n}{2}\{2a + (n - 1)d\}$, we get

$$190 = \frac{n}{2}\{2 \times 1 + (n - 1) \times 1\} \Rightarrow 380 = 2n + n^2 - n.$$

$\Rightarrow n^2 + n - 380 = 0 \Rightarrow (n + 20)(n - 14) = 0$

$\Rightarrow n = 19$ rejective – 20 days.

Or

The total money saved = $\frac{19}{2}\{2 \times 1 + 18 \times 1\} \times 5$ (Because of 5 rupee coins)

$$= \frac{19 \times 20}{2} \times 5 = ₹\ 190 \times 5 = ₹\ 950.$$

37. (*i*) Coordinates of P are (3, 2), with A as (0, 0).

(*ii*) Coordinates of Q are (6, 4), with A as (0, 0).

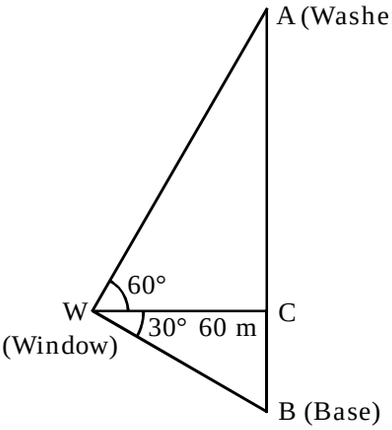
(*iii*) Coordinates of mid-point of PQ = $\left(\frac{3 + 6}{2}, \frac{2 + 4}{2}\right) = \left(\frac{9}{2}, 3\right)$.

Or

With B as origin, coordinates of Q are (12 – 6, 4) = (6, 4) and that of R are (12 – 4, 5) = (8, 5).

So, mid-point of QR = $\left(\frac{6+8}{2}, \frac{4+5}{2}\right) = \left(7, \frac{9}{2}\right)$.

38. (*i*) Labelled diagram is as shown below :



(*ii*) Let BC = *h*.

So, from ΔBWC, $\frac{BC}{WC} = \tan 30° \Rightarrow \frac{BC}{60} = \frac{1}{\sqrt{3}} \Rightarrow BC = \frac{60}{\sqrt{3}}$ m $= 20\sqrt{3}$ m.

So, window is $20\sqrt{3}$ m above the ground.

(*iii*) From ΔAWC, $\frac{AC}{WC} = \tan 60° \Rightarrow AC = WC\sqrt{3} = 60\sqrt{3}$ m.

Hence, AB = AC + BC = $60\sqrt{3} + 20\sqrt{3} = 80\sqrt{3}$ m.

Or

From ΔAWC, $\frac{AC}{AW} = \sin 60° \Rightarrow \frac{60\sqrt{3}}{AW} = \frac{\sqrt{3}}{2} \Rightarrow AW = \frac{60\sqrt{3} \times 2}{\sqrt{3}} = 120$ m.

So, distance between Ranjan and washers is 120 m.

3 SAMPLE QUESTION PAPER

MATHEMATICS (Standard)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions : Same as in CBSE Sample Question Paper.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then LCM (a, b) is.

(A) xy (B) xy^2 (C) x^3y^3 (D) x^2y^2

2. How many prime factors are there in prime factorisation of 5005 ?

(A) 2 (B) 4 (C) 6 (D) 7

3. Which is the best method to find the roots of the equation $2x^2 + 7x + 5 = 0$?

(A) Splitting the middle term $7x$ to $5x + 2x$ and then factorizing the equation

(B) Splitting the middle term $7x$ to $10x – 3x$ and then factorizing the equation

(C) Using the method of completing the square to get $x^2 + \frac{7}{2}x + 5 = 0$

(D) Using the method of completing the square to get $x^2 + 7x + 5 = 0$

4. Shivi gave a note of ₹ 2,000 for a pair of jeans worth ₹ 500. She was returned 11 notes in denominations of ₹ 200 and ₹ 100. Which pair of equations can be used to find the number of ₹ 200 notes, x, and the number of ₹ 100 notes y? How many notes of ₹ 200 did she get?

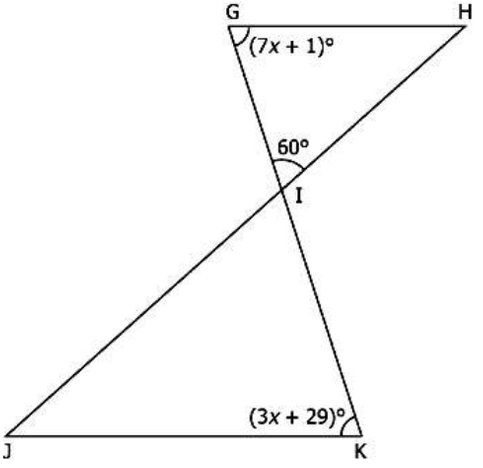
(A) $x + y = 11$ and $200x + 100y = 1500$; 8 (B) $y = x + 11$ and $200x + 100y = 2000$; 4

(C) $x + y = 11$ and $200x + 100y = 1500$; 4 (D) $x + y = 11$ and $100x + 200y = 2000$; 10

5. $(x^2 + 1)^2 – x^2 = 0$ has

(A) four real roots (B) two real roots (C) no real roots (D) one real root

6. Consider the figure.



Which of the following information help proving that triangle GHI is similar to triangle KIJ?

Information 1 : $\angle H = 70°$

Information 2 : $\angle J = 70°$

(A) Each information alone is sufficient.

(B) Information (1) alone is sufficient, but Information (2) alone is not sufficient.

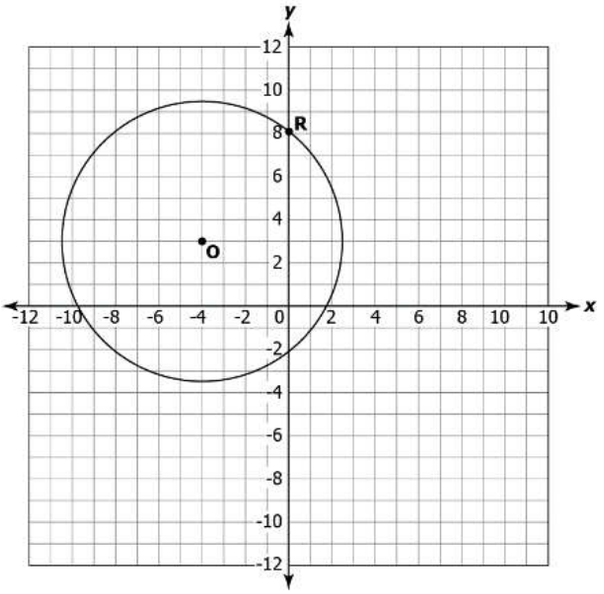
(C) Information (2) alone is sufficient, but Information (1) alone is not sufficient.

(D) Both information together is sufficient, but neither information alone is sufficient.

7. In ΔABC, DE || AB. If CD = 3 cm, EC = 4 cm, BE = 6 cm then, DA is equal to _____.

(A) 7.5 cm (B) 3 cm (C) 4.5 cm (D) 6 cm

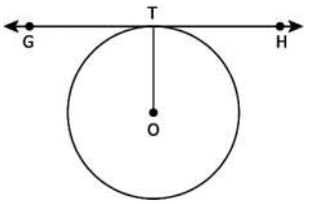
8. The graph of a circle with centre O with point R on its circumference is shown.



What is the side length of the square that circumscribes the circle?

(A) $2\sqrt{41}$ units (B) $\sqrt{41}$ units (C) $3\sqrt{17}$ units (D) $6\sqrt{17}$ units

9. A circle with centre O is shown below.



Which of the following statements is true?

(A) There can be only one line passing through point T such that it is parallel to OT.

(B) There can be only one line passing through point T such that it is perpendicular to OT.

(C) There can be any number of lines passing through point T such that they are parallel to OT.

(D) There can be any number of lines passing through point T such that they are perpendicular to OT.

10. If $\sin A = \frac{1}{2}$, then the value of cot A is

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1

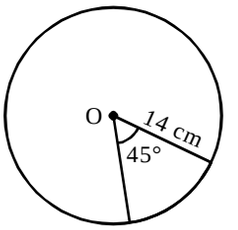
11. In ΔABC, $\angle C = 90°$, then the value of $\cos^2 A + \cos^2 B$ is :

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

12. If $\cos A = \frac{24}{25}$, the value of sin A is

(A) $\frac{25}{24}$ (B) $\frac{7}{25}$ (C) $\frac{7}{5}$ (D) $\frac{5}{7}$

13. An arc of a circle of radius 14 cm, subtends an angle of 45° at the centre as shown :



Which of these options is correct?

(A) The arc shown is a major arc and its length is 38.5 cm.

(B) The arc shown is a minor arc and its length is 11 cm.

(C) The arc shown is a minor arc and its length is 5.5 cm.

(D) The arc shown is a major arc and its length is 77 cm.

14. Two identical solid cubes of side k units are joined end to end. What is the volume, in cubic units, of the resulting cuboid?

(A) k^3 (B) $2k^3$ (C) $3k^3$ (D) $6k^3$

15. A vendor sells glasses of juice, which is in the form of a cylinder mounted by a hollow hemisphere. The diameter of the hemisphere and the cylinder is 8.4 cm. If the total height of the glass is 15 cm, which of these is closest to the volume, in cubic centimeters, of 8 such glasses ? $\left(\text{Use } \pi = \frac{22}{7}\right)$

(A) 599 cm^3 (B) 754 cm^3 (C) 6032 cm^3 (D) 29083 cm^3

16. Upper limit of the modal class for the data

Class	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	7	5	8	6	7	6

(A) 40 (B) 30 (C) 60 (D) 70

17. Lower limit of the modal class for the data

Class	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
Frequency	15	12	8	6	14

(A) 25 (B) 30 (C) 20 (D) 40

18. To win a price in a game, you need to first choose one of the 4 doors, 1, 2, 3, 4 and then need to choose one of the three boxes A, B, C and then need to choose between two colours red and green. How many of the possible outcomes of this game include selecting Box A and red colour?

(A) 4 (B) 8 (C) 12 (D) 24

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. **(A) :** HCF of 95 and 152 is 19.
(R) : LCM of 95 and 152 is 95 × 152.

20. **(A) :** Point p dividing the line-segment joining the points A(2, 1) and B(5, –8) in the ratio 1 : 3 is $\left(\frac{11}{4}, \frac{-5}{4}\right)$.

(R) : The coordinates of the point $p(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_1+, m_2x_2}{m_1 + m_2}, \frac{m_1y_1+, m_2y_2}{m_1 + m_2}\right)$.

SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. Given the linear equation $3x + 4y = 9$, write another linear equation in these two variables such that the geometrical representation of the pair so formed is :
(*i*) intersecting lines, (*ii*) coincident lines.

22. If the perimeters of two similar triangles ABC and DEF are 50 cm and 70 cm respectively and one side of ΔABC = 20 cm, then find the corresponding side of ΔDEF.

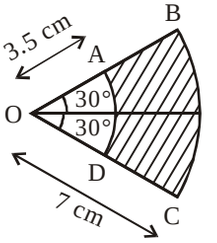
Or

A vertical pole of length 8 m casts a shadow 6 m long on the ground and at the same time a tower casts a shadow 30 m long. Find the height of tower.

23. Two concentric circles of radii 5 cm and 3 cm are given. Find the length of the chord of the larger circle which touches the smaller circle.

24. In a right angled ΔUVW right angled at W, if sin U = sin V, then show that $\angle U = \angle V$.

25. In the given figure, there are sectors of two concentric circles of radii 7 cm and 3.5 cm respectively. Find the area of the shaded portion.



Or

An exhaust fan has four blades of same size with radius of each blade 20 cm as shown in the figure. Find the area of each blade, if they all are equally spaced. (Take $\pi = 3.14$)

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they all change simultaneously at 8 : 00 hours, then at what time will they again change simultaneously ?

27. Let α and β be zeroes of a quadratic polynomial $x^2 - 3$. Then, form a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

28. Solve by substitution :

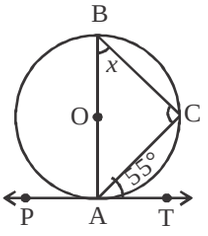
$$x - \frac{y}{2} = 3$$

$$\frac{x}{2} - \frac{2y}{3} = \frac{2}{3}$$

Or

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay ₹ 3000 as hostel charges, whereas Mansi who takes food for 25 days pays ₹ 3500 as hostel charges. Find the fixed charges and the cost of food per day.

29. Find the value of x in the given figure, if PAT is a tangent to the circle with centre O at point A and AOB is a diamter.



30. If $\sec\theta = 2$, then evaluate $\dfrac{4\cos\theta - \sqrt{3}\sin\theta}{\tan\theta - \cot\theta}$.

Or

Given $\tan A = \dfrac{5}{12}$, find the other trigonometric ratios of the angle A.

31. A pair of dice is thrown once. What is the probability of getting the number 4 on one die at least ?

SECTION – D

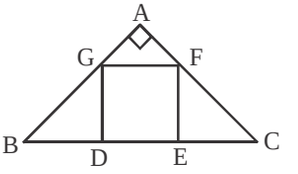
Section-D consists of 4 questions of 5 marks each.

32. Express $\left(\dfrac{4x-3}{2x+1}\right) - 10\left(\dfrac{2x+1}{4x-3}\right) = 3, \left(x \neq \dfrac{-1}{2}, x \neq \dfrac{3}{4}\right)$ in standard form and then find its roots by factorisation method.

Or

The taxi charges in a city consists of a fixed charge together with the charge for the distance covered. For a distance of 8 km, the charge paid is ₹ 111 and for a journey of 12 km, the charge paid is ₹ 159. What are the fixed charges and the charge per km ? How much does a person have to pay for travelling a distance of 5 km ?

33. In the figure, ΔEFG is a square and $\angle BAC = 90°$. Prove that :



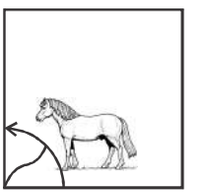
(i) $\Delta AGF \sim \Delta DBG$

(ii) $\Delta AGF \sim \Delta EFC$

(iii) $\Delta DBG \sim \Delta EFC$

(iv) $DE^2 = BD \times EC$

34. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find :



(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area, if the rope were 10 m long instead of 5 m. ($\pi = 3.14$)

Or

A container opened from the top is made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively.

(i) Find the cost of the milk which can completely fill the container at the rate of ₹ 20 per litre.

(ii) Also, find the cost of metal sheet used to make the container, if it costs ₹ 8 per 100 cm^2. (Use $\pi = 3.14$)

35. Find the mean and mode for the following data :

Class	Frequency
10 – 20	4
20 – 30	8
30 – 40	10
40 – 50	12
50 – 60	10
60 – 70	4
70 – 80	2

SECTION – E

Case study based questions are compulsory.

36. Aditya purchased a car and he took loan from a bank for his car under Car loan offer during Festivals. He repays his total loan of ₹ 1,18,000 by paying every month starting with the first instalment of ₹ 1000, if he increases the instalment by ₹ 100 every month, answer the following questions :

(*i*) Write an AP for the above situation.

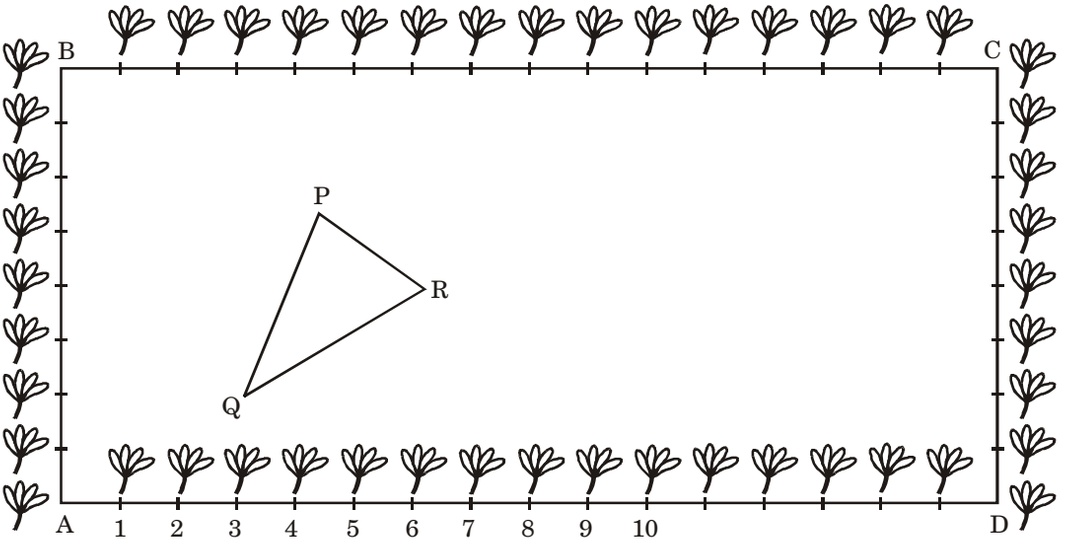
(*ii*) What amount will be paid by him in the 30th instalment?

(*iii*) What amount of loan does he still have to pay after the 30th instalment?

Or

What amount will be paid by him in the 30th instalment?

37. The class X students school in Krishnagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(*i*) Taking A as origin, find the coordinates of R.

(*ii*) What will be the coordinates of R, if C is the origin ?

(*iii*) Taking A as origin, what is the mid-point of R ?

Or

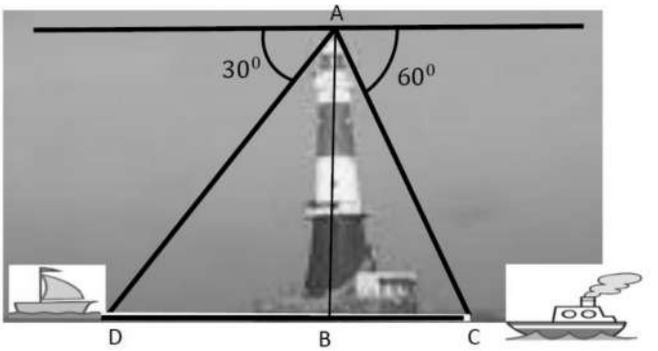
What is the distance between Q and R ?

38. A lighthouse is a tall tower with light near the top. These are often built on islands, coasts or on cliffs. Lighthouses on water surface act as a navigational aid to the mariners and send warning to boats and ships for dangers. Initially wood, coal would be used as illuminators. Gradually it was replaced by candles, lanterns, electric lights. Nowadays they are run by machines and remote monitoring. Prongs Reef lighthouse of Mumbai was constructed in 1874-75. It is approximately 40 metres high and its beam can be seen at a distance of 30 kilometres. A ship and a boat are coming towards the lighthouse from opposite directions. Angles of depression of flash light from the lighthouse to the boat and the ship are 30° and 60° respectively (see figure).

(*i*) Find the distance of boat from the lighthouse.

(*ii*) Find the distance of ship from the lighthouse.

(*iii*) Find the time taken by the boat to reach the light house if it is moving at the rate of 2 km per hour.



Or

Find the time taken by the ship to reach the lighthouse, if it is moving at the rate of 3 km per hour.

Answers

1. (D) x^2y^2 **2.** (B) 4

3. (A) Splitting the middle term $7x$ to $5x + 2x$ and then factorizing the equation

4. (C) $x + y = 11$ and $200x + 100y = 1500$; 4

5. (D) one real root **6.** (A) Each information alone is sufficient. **7.** (C) 4.5 cm

8. (A) $2\sqrt{41}$ units

9. (B) There can be only one line passing through point T such that it is perpendicular to OT.

10. (A) $\sqrt{3}$ **11.** (A) 1 **12.** (B) $\frac{7}{25}$

13. (B) The arc shown is a minor arc and its length is 11 cm. **14.** (B) $2k^3$

15. (C) 6032 cm^3 **16.** (A) 40 **17.** (C) 20 **18.** (A) 4

19. (C) (A) is true but (R) is false. **20.** (C) (A) is true but (R) is false.

21. (*i*) $5x - 9y = 11$ (*ii*) $9x + 12y = 27$

22. Perimeters of the two similar triangles are 50 cm and 70 cm.

$\therefore$ Ratio of their corresponding sides = $\frac{50}{70} = \frac{5}{7}$.

Now, in ΔABC, let us suppose that AB = 20 cm.

Then, $\frac{AB}{DE} = \frac{5}{7} \Rightarrow \frac{20}{DE} = \frac{5}{7}$

(where DE is the corresponding side of ΔDEF)

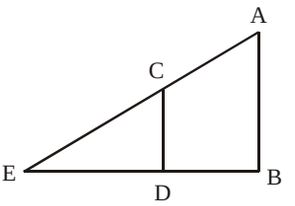
$\Rightarrow$ $DE = \frac{20 \times 7}{5} = 28$ cm.

Thus, the side of ΔDEF corresponding to side 20 cm of ΔABC is 28 cm.

Or

Let CD be the vertical pole and AB be the tower.
So, CD = 8 m.
DE and BE are shadows of the pole and tower respectively.
So, DE = 6 m and BE = 30 m.

Now, $\Delta ABE \sim \Delta CDE$ (By AA similarity criterion)



$\Rightarrow$ $\frac{AB}{CD} = \frac{BE}{DE}$.

$\Rightarrow$ $\frac{AB}{8} = \frac{30}{6}$

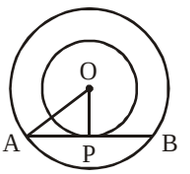
$\Rightarrow$ $AB = \frac{30 \times 8}{6}$

$= 40$ m.

So, height of the tower is 40 m.

23. ***Hint :*** The fact that APB is tangent to the smaller circle will be used to show that OP $\perp$ AB. Then, the use of Pythagoras. Theorem will give the answer.

Let the chord AB of the larger circle touches the smaller circle at P.



Here, OA = 5 cm and OP = 3 cm.

So, from rt. ΔOAP, $OA^2 = OP^2 + AP^2$ ($\angle OPA = 90°$)

$\Rightarrow$ $(5)^2 = (3)^2 + AP^2$.

$\Rightarrow$ $AP^2 = 25 - 9 = 16$

$\Rightarrow$ $AP = \sqrt{16} = 4$ cm.

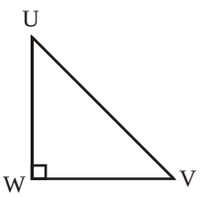
So, $AB = 2AP$ (OP bisects the chord AB)

$= 2 \times 4$ cm

$= 8$ cm.

24. In ΔUVW, where $\angle W = 90°$,

we are given $\sin U = \sin V$.



$\Rightarrow$ $\frac{VW}{UV} = \frac{UW}{UV}$

$\Rightarrow$ $VW = UW$

So, $\angle U = \angle V$.

(Angles opposite the equal sides)

25. Area of the shaded region $= \frac{60°}{360°} \times \frac{22}{7}(7^2 - 3.5^2)$ cm^2.

$= \frac{1}{6} \times \frac{22}{7} \times 3.5 \times 10.5$ cm^2.

$= \frac{22 \times 35 \times 105}{6 \times 7 \times 100}$ cm^2.

$= \frac{11 \times 5 \times 35}{100}$ cm^2

$= 19.25$ cm^2.

Or

$$\text{Area of each blade} = \frac{D}{360°} \times \pi \times 20^2 \text{ cm}^2.$$

$$= \frac{45°}{360°} \times 3.14 \times 400 \text{ cm}^2. \qquad \left[D = \frac{360°}{8} = 45°\right]$$

$$= \frac{3.14 \times 400}{8} \text{ cm}^2 = 3.14 \times 50 \text{ cm}^2.$$

$$= 157 \text{ cm}^2.$$

26. ***Hint :*** By using LCM of 48, 72 and 108

We have : $48 = 2 \times 2 \times 2 \times 2 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

and $108 = 2 \times 2 \times 3 \times 3 \times 3.$

So, $\text{LCM } (48, 72, 108) = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 432.$

Now, 432 seconds = 7 minutes and 12 seconds.

Hence, they will again change simultaneously at 8 : 07 : 12 hours.

27. α and β are zeroes of the polynomial $x^2 - 3$, *i.e.*, $x^2 - 0x - 3$.

So, $\alpha + \beta = \frac{0}{1} = 0$

and $\alpha\beta = -\frac{3}{1} = -3.$

Now, $\text{sum of } \frac{1}{\alpha} \text{ and } \frac{1}{\beta} = \frac{1}{\alpha} + \frac{1}{\beta}$

$$= \frac{\alpha + \beta}{\alpha\beta} = \frac{0}{-3} = 0$$

and $\text{product of } \frac{1}{\alpha} \text{ and } \frac{1}{\beta} = \frac{1}{\alpha} \times \frac{1}{\beta}$

$$= \frac{1}{\alpha\beta} = \frac{1}{-3} = -\frac{1}{3}.$$

So, required polynomial is $x^2 - (\text{Sum of zeroes})x + (\text{Product of zeroes})$

$$= x^2 - 0 \times x + \left(-\frac{1}{3}\right) = x^2 - \frac{1}{3}.$$

It can be represented as $3x^2 - 1$ also.

28. We have :

$$x - \frac{y}{2} = 3 \qquad \ldots(1)$$

and $$\frac{x}{2} - \frac{2y}{3} = \frac{2}{3} \qquad \ldots(2)$$

From (1), $x = 3 + \frac{y}{2} = \frac{6 + y}{2}.$

Substituting this value of x in (2), we get

$$\frac{1}{2} \times \frac{6 + y}{2} - \frac{2y}{3} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{6 + y}{4} - \frac{2y}{3} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{12(6+y)}{4} - \frac{12\times 2y}{3} = 12 \times \frac{2}{3}$$

$$\Rightarrow \quad 18 + 3y - 8y = 8$$

$$-5y = 8 - 18$$

$$\Rightarrow \quad -5y = -10$$

$$\Rightarrow \quad y = \frac{-10}{-5} = 2.$$

So, $x = 3 + \frac{y}{2} = 3 + \frac{2}{2} = 4.$

Thus, the solution is $x = 4$ and $y = 2$.

Or

Let the fixed monthly charges be ₹ x and the cost of food per day be ₹ y. So, according to given conditions, we get

$$x + 20y = 3000 \qquad ...(1)$$

and $$x + 25y = 3500 \qquad ...(2)$$

From eqn. (1), $x = 3000 - 20y$.

Substituting $x = 3000 - 20y$ in eqn. (2), we get

$$3000 - 20y + 25y = 3500 \quad \Rightarrow \quad y = \frac{500}{5} = 100.$$

Putting $y = 100$ in eqn. (1), we get

$$+ 20 \times 100 = 3000$$

$$\Rightarrow \quad x + 2000 = 3000$$

$$\Rightarrow \quad x = 3000 - 2000 = 1000.$$

Thus, fixed charges are ₹ 1000 and cost of food per day is ₹ 100.

29. We know that $\angle OAT = 90°$ (Angle between radius and tangent)

$$\Rightarrow \quad \angle OAC + \angle CAT = 90°.$$

$$\Rightarrow \quad \angle OAC + 55° = 90°$$

$$\Rightarrow \quad \angle OAC = 90° - 55° = 35°.$$

Also, $\angle ACB = 90°$ (Angle in a semicircle)

Now, in rt ΔABC

$$\angle ABC + \angle BAC + \angle ACB = 180°.$$

So, $$x + 35° + 90° = 180°$$

$$\Rightarrow \quad x = 180° - 125°$$

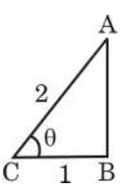
$$= 55°.$$

30. We have :

$$\sec\theta = 2.$$

$$\Rightarrow \quad \frac{1}{\cos\theta} = 2$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}.$$



Therefore, if in ΔABC, $BC = 1$ and $AC = 2$, then

$$AB = \sqrt{4-1} = \sqrt{3}.$$

So, $$\sin\theta = \frac{\sqrt{3}}{2}.$$

$\Rightarrow \quad \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sqrt{\frac{3}{2}}}{\frac{1}{2}} = \dfrac{\sqrt{3}}{2} \times \dfrac{2}{1} = \sqrt{3}$.

and $\quad \cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{2} \times \dfrac{2}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$.

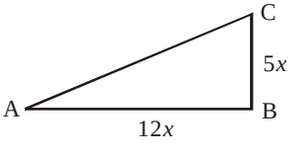
Therefore, $\dfrac{4\cos\theta - \sqrt{3}\sin\theta}{\tan\theta - \cot\theta} = \dfrac{4 \times \frac{1}{2} - \sqrt{3} \times \frac{\sqrt{3}}{2}}{\sqrt{3} - \frac{1}{\sqrt{3}}}$

$= \dfrac{2 - \frac{3}{2}}{\frac{3-1}{\sqrt{3}}} = \dfrac{\frac{4-3}{2}}{\frac{2}{\sqrt{3}}}$

$= \dfrac{1}{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4}$.

Or

$\tan A = \dfrac{5}{12}$.



So, if $BC = 5x$,
then $AB = 12x$.
Hence, $AC^2 = AB^2 + BC^2$
$= (12x)^2 + (5x)^2$
$= 144x^2 + 25x^2 = 169x^2$.

$\Rightarrow \quad AC = \sqrt{169x^2} = 13x$.

So, we have :

$\sin A = \dfrac{BC}{AC} = \dfrac{5x}{13x} = \dfrac{5}{13}$, $\cos A = \dfrac{AB}{AC} = \dfrac{12x}{13x} = \dfrac{12}{13}$,

$\text{cosec}\, A = \dfrac{AC}{BC} = \dfrac{13x}{5x} = \dfrac{13}{5}$, $\sec A = \dfrac{AC}{AB} = \dfrac{13x}{12x} = \dfrac{13}{12}$

and $\cot A = \dfrac{AB}{BC} = \dfrac{12x}{5x} = \dfrac{12}{5}$.

31. Total outcomes = 6 × 6 = 36.
Favourable outcomes for number 4 at least on one die are (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (1, 4), (2, 4), (3, 4), (5, 4) and (6, 4) (Total 11).
So, required probability = $\dfrac{11}{36}$.

[***Check :*** May write $\dfrac{10}{36}$ ignoring (4, 4).]

32. ***Hint :*** Quadratic equation will be formed after simplification.

Let $\dfrac{4x - 3}{2x + 1} = y$.

So, the given equation becomes

$$y - 10 \times \frac{1}{y} = 3.$$

$\Rightarrow \quad y^2 - 10 = 3y$

$\Rightarrow \quad y^2 - 3y - 10 = 0.$

$\Rightarrow \quad y^2 - 5y + 2y - 10 = 0$

$\Rightarrow \quad y(y - 5) + 2(y - 5) = 0$

$\Rightarrow \quad (y - 5)(y + 2) = 0$

$\Rightarrow \quad y = 5 \quad \text{or} \quad y = -2.$

Taking, $y = 5$, we get

$$\frac{4x - 3}{2x + 1} = 5$$

$\Rightarrow \quad 4x - 3 = 10x + 5.$

$\Rightarrow \quad -6x = 8 \quad \Rightarrow \quad x = \frac{-8}{6} = \frac{-4}{3}.$

Taking, $y = -2$, we get

$$\frac{4x - 3}{2x + 1} = -2$$

$\Rightarrow \quad 4x - 3 = -4x - 2.$

$\Rightarrow \quad 8x = 1 \quad \Rightarrow \quad x = \frac{1}{8}.$

So, solutions are $x = \frac{-4}{3}$ and $x = \frac{1}{8}$.

Or

Let us assume that the fixed charge of the taxi be x and the charge per kilometre be y. Then, according to the question,

$$x + 8y = 111 \qquad \ldots(1)$$

and $$x + 12y = 159 \qquad \ldots(2)$$

Subtracting equation (1) from (2), we get

$$\begin{array}{rcl} x + 12y &=& 159 \\ x + 8y &=& 111 \\ - \quad - & & - \\ \hline 4y &=& 48 \end{array}$$

or $$y = \frac{48}{4} = 12.$$

Putting $y = 12$ in equation (1), we get

$$x + 8 \times 12 = 111$$

or $$x = 111 - 96 = 15.$$

Hence, the fixed charge is ₹ 15 and charge per km is ₹ 12.

Payment for travelling 5 km $= 15 + 5 \times 12$

$= 15 + 60 =$ ₹ 75.

33. **Given :** In the given figure, ΔABC is right angled at A.

$\Rightarrow \quad \angle BAC = 90°$ and DEFG is a square.

To prove :

(*i*) $\Delta AGF \sim \Delta DBG$

(*ii*) $\Delta AGF \sim \Delta EFC$

(*iii*) $\Delta DBG \sim \Delta EFC$

(*iv*) $DE^2 = BD \times EC$

Proof : In the figure, GD $\perp$ BC and FE $\perp$ BC, and GF $\parallel$ BC.

(*i*) In ΔAGF and ΔDBG,

$\angle GAF = \angle BDG$ [Each 90°]

$\angle AGF = \angle GBD$ [Corresponding angles of GF || BC]

So, $\Delta AGF \sim \Delta DBG$ ...(*i*)

[AA similarity]

(*ii*) In ΔAGF and ΔEFC,

$\angle GAF = \angle FEC$ [Each 90°]

$\angle AFG = \angle ECF$ [Corresponding angles of GF || BC]

So, $\Delta AGF \sim \Delta EFC$ ...(*ii*)

[AA similarity]

(*iii*) Since $\Delta AGF \sim \Delta DBG$ [From (*i*)]

and $\Delta AGF \sim \Delta EFC$ [From (*ii*)]

therefore, $\Delta DBG \sim \Delta EFC$ [Triangles similar to same triangle are similar to each other]

(*iv*) Since $\Delta DBG \sim \Delta EFC$, therefore

$$\frac{DG}{BD} = \frac{EC}{EF}$$ [Corresponding sides]

or $DG \times EF = BD \times EC$

But $DG = DE = EF$ [Sides of a square]

$\Rightarrow$ $DE \times DE = BD \times EC.$

or $DE^2 = BD \times EC.$

Hence, proved.

34. (*i*) Area of the part of the field in which horse can graze

$$= \frac{90°}{360°} \times \pi \times 5^2 \text{ m}^2$$

$$= \frac{1}{4} \times \pi \times 25 \text{ m}^2$$

$$= \frac{25}{4} \times 3.14 \text{ m}^2$$

$$= 19.625 \text{ m}^2.$$

(*ii*) Increase in area, if rope is 10 m instead of 5 m

$$= \frac{90}{360} \times \pi(10^2 - 5^2).$$

$$= \frac{1}{4} \times 3.14 \times (100 - 25) \text{ m}^2.$$

$$= \frac{75}{4} \times 3.14 \text{ m}^2.$$

$$= 58.875 \text{ m}^2.$$

Or

(*i*) Volume of the container $= \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$$= \frac{3.14 \times 16}{3}(400 + 64 + 20 \times 8) \text{ cm}^3$$

$$= \frac{3.14 \times 16}{3}(624) \text{ cm}^3 = \frac{3.14 \times 16 \times 624}{3 \times 1000} \text{ litres}$$

$$= \frac{314 \times 16 \times 624}{300 \times 1000} \text{ litres}$$

So, $\text{cost of milk} = ₹\ 20 \times \frac{314 \times 16 \times 624}{300 \times 1000}$

$= ₹\ \frac{314 \times 16 \times 208}{5000} = ₹\ 209$

(*ii*) Surface area of the container $= \pi (r_1 + r_2) \sqrt{h^2 + (r_1 - r_2)^2} + \pi r_2^2$

$= 3.14\ (20 + 8) \sqrt{256 + (12)^2} + 3.14 \times 64$

$= 3.14\ \{28 \sqrt{400} + 64\}\ \text{cm}^2$

$= 3.14\ (28 \times 20 + 64)\ \text{cm}^2$

$= 3.14 \times (560 + 64)\ \text{cm}^2$

$= 3.14 \times 624\ \text{cm}^2$

So, $\text{cost of metal} = ₹\ 3.14 \times 624 \times \frac{8}{100}$

$= ₹\ \frac{314 \times 624 \times 8}{10000} = ₹\ \frac{314 \times 624}{1250} = ₹\ 156.75$

35. The required table for finding mean and mode is as follow :

Classes	Class mark (x_i)	Frequency (f_i)	$f_i x_i$
10 – 20	15	4	60
20 – 30	25	8	200
30 – 40	35	10	350
40 – 50	45	12	540
50 – 60	55	10	550
60 – 70	65	4	260
70 – 80	75	2	150
Total		50	2110

So, mean $= \frac{\sum f_i x_1}{\sum f_i} = \frac{2110}{50} = 42.2.$

Now, maximum frequency is 12. So, the modal class is 40 – 50.

Therefore, $l = 40$, f_1, 12, $f_0 = 10$, $f_2 = 10$, $h = 10$.

Hence, $\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h.$

$= 40 + \left(\frac{12 - 10}{2 \times 12 - 10 - 10} \right) \times 10.$

$= 40 + 5 = 45.$

36. (*i*) AP is 1000, 1100, 1200, ...

(*ii*) Amount paid in 30th instalment = 1000 + 29 × 100 = ₹ 3900.

(*iii*) Amount to be paid after 30th instalment = ₹ (118000 – 73500) = ₹ 44500.

Or

Amount paid upto 30th instalment $= \frac{30}{2}\{2 \times 1000 + 29 \times 100\}$

$= 15 \times (4900)$

$= ₹\ 73500.$

37. (*i*) Coordinates of R are (6, 4), with A as origin.

(*ii*) If C is the origin, coordinates of R will be (16 – 6, 8 – 4) = (10, 4).

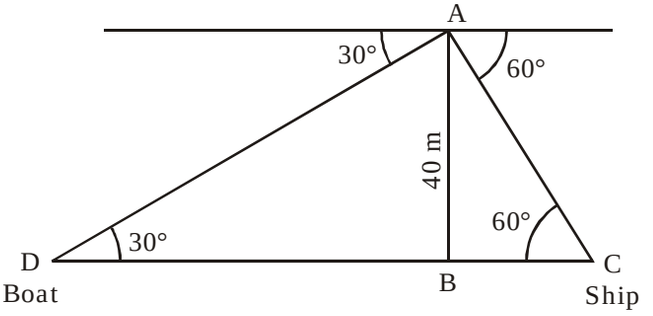
(*iii*) Coordinates of Q are (3, 2) and that of R are (6, 4) with A as origin.

So, required mid-point $= \left(\frac{3+6}{2}, \frac{2+4}{2}\right) = \left(\frac{9}{2}, 3\right)$.

Or

QR $= \sqrt{(6-3)^2 + (4-2)^2} = \sqrt{9+4} = \sqrt{13}$ units.

38. (*i*) From ΔABD, $\frac{AB}{BD} = \tan 30° \Rightarrow \frac{40}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 40\sqrt{3}$ m.



Thus, distance of the boat from lighthouse is $40\sqrt{3}$ m.

(*ii*) From ΔABC, $\frac{AB}{BC} = \tan 60° \Rightarrow \frac{40}{BC} = \sqrt{3} \Rightarrow BC = \frac{40}{\sqrt{3}}$ m $= \frac{40\sqrt{3}}{3}$ m.

So, distance of the ship from lighthouse is $\frac{40\sqrt{3}}{3}$ m.

(*iii*) Time taken by the boat to reach the lighthouse with a speed of 2 km per hour

$= \frac{40\sqrt{3}}{2000}$ hours $= \frac{\sqrt{3}}{50}$ hours $= \frac{6\sqrt{3}}{5}$ minutes.

Or

Time taken by the ship to reach the lighthouse with a speed of 3 km per hour

$= \frac{40\sqrt{3} \times 60}{3 \times 3000}$ minutes $= \frac{40\sqrt{3}}{3 \times 50} = \frac{4\sqrt{3}}{15}$ minutes.

4 SAMPLE QUESTION PAPER

MATHEMATICS (Standard)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions : Same as in CBSE Sample Question Paper.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. $119^2 - 111^2$ is :

(A) Prime number (B) Composite number

(C) An odd prime number (D) An odd composite number

2. A teacher creates the question, "Which of the following could be the sum of two rational numbers?"

She now needs to create three incorrect choices and one correct answer. Which option shows the choices that the teacher should create?

(A) First choice : π; Second choice : 20 + 16; Third choice : 50-1; Correct Answer : 49

(B) First choice : 227; Second choice : 25 + 16; Third choice : 64; Correct Answer : 5

(C) First choice : 125; Second choice : 36 + 42; Third choice : 81; Correct Answer : 169

(D) None of them

3. If no roots of the equation $x^2 - px + 1 = 0$ is real, then

(A) $p > 2$ (B) $p < -2$ (C) $p = 2$ (D) $-2 < p < 2$

4. Consider the equations shown :

$$4x + 3y = 41$$
$$x + 3y = 26$$

Which of these is the correct way of solving the given pair of equations?

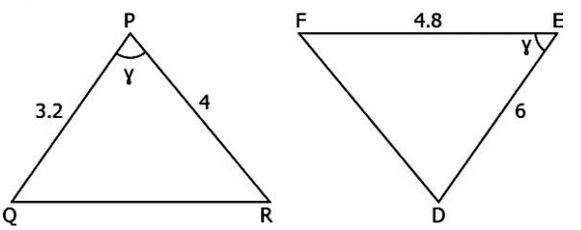
(A) $4x - x + 3y - 3y = 41 - 26$ (B) $4(x + 3y) + 3y = 41$

(C) $4x + x + 3y - 3y = 41 - 26$ (D) $4x + 3y + 3y = 41 - 26$

5. The discriminant of quadratic equation $x^2 - 4x + 1 = 0$ is

(A) 4 (B) 8 (C) 12 (D) 16

6. Observe the two triangles shown below.



Which statement is correct?

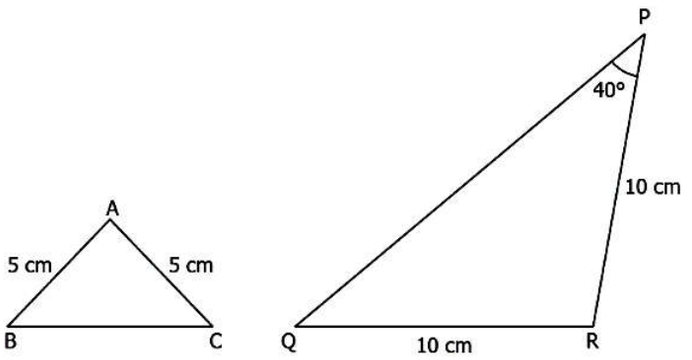
(A) Triangles are similar by SAS.

(B) Triangles are similar by SSA.

(C) Triangles are not similar as sides are not in proportion.

(D) No valid conclusion about similarity of triangles can be made as angle measures are not known.

7. Consider the triangles below.



Which statement is correct?

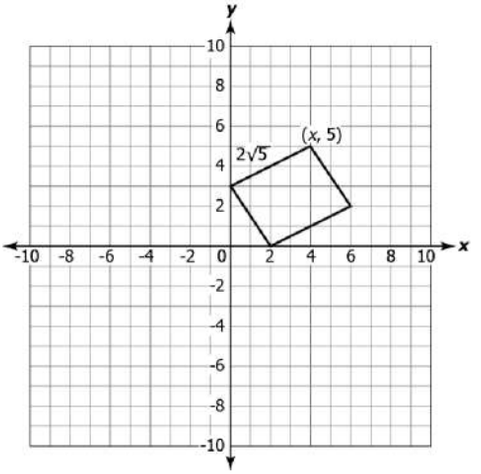
(A) Triangles are similar as all isosceles triangles are similar.

(B) Triangles are similar as corresponding sides of the triangles are in the ratio 1 : 2.

(C) For triangles to be similar, the measure of $\angle A = 40°$.

(D) For triangles to be similar, the measure of $\angle A = 100°$.

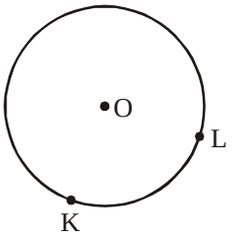
8. The figure shows a parallelogram with one of its vertices intersecting the y-axis at 3 and another vertex intersecting the x-axis at 2.



If (m, n) is the intersection point of the diagonals of the parallelogram, which relation is correct?

(A) $m = 0.5 + n$ (B) $m = n - 0.5$ (C) $m = 1.50 + n$ (D) $m = n - 1.50$

9. A circle is shown below.



Statement I : There is only one line passing through point K which makes an angle of 90° with OK.

Statement II : The shortest distance of a tangent passing through point L from the centre O is equal to the radius of the circle, OL.

Statement III : One tangent can pass through two points K and L of a circle.

Which statement(s) is/are correct?

(A) Statement I and II (B) Statement I and III

(C) Statement II and III (D) Statement I, II and III

10. Given that $\sin\theta = \frac{a}{b}$, then $\tan\theta =$

(A) $\frac{b}{\sqrt{b^2 - a^2}}$ (B) $\frac{\sqrt{b^2 - a^2}}{b}$ (C) $\frac{a}{\sqrt{b^2 - a^2}}$ (D) $\frac{\sqrt{b^2 - a^2}}{a}$

11. Which of the following is not defined ?

(A) cos 0° (B) tan 45° (C) sec 90° (D) sin 90°

12. The value of $\frac{\tan 45°}{\sin 45° + \cos 45°}$ is :

(A) $\frac{1}{\sqrt{2}}$ (B) 1 (C) $\frac{1}{2}$ (D) $\sqrt{2}$

13. Abhinav made a model for his school project in the shape of a cylinder of radius 7 cm and height 21 cm, with hemisphere surmounted on one end. He wants to cover the entire model with decorative paper. What is the area, in centimeter square, of paper that is required to cover the model ? $\left(\text{Use } \pi = \frac{22}{7}\right)$

(A) 1386 cm^2 (B) 1232 cm^2 (C) 1012 cm^2 (D) 976 cm^2

14. The ratio of the volumes of two spheres is 27 : 64. If r_1 and r_2 are the radii of these two spheres respectively, then $r_1 : (r_2 - r_1)$ is

(A) 3 : 4 (B) 1 : 3 (C) 3 : 1 (D) 4 : 3

15. The ratio of the surface areas of two cubes is 4 : 9, then volumes are in the ratio

(A) 8 : 27 (B) 27 : 8 (C) 9 : 4 (D) 16 : 27

16. A grouped data is shown below :

Class Interval	Frequency
0-15	2
15-30	26
30-45	32
45-60	42
60-75	28
75-90	30

Which of the following is the most effective measure of central tendency?

(A) Mean because the data has extreme data points.

(B) Median because the data has extreme data points.

(C) Mean because the data has no extreme data points.

(D) Median because the data has no extreme data points.

17. The table below shows the age of people attending a musical concert.

Age (*in years*)	Number of people
0-10	2
10-20	19
20-30	32
30-40	23
40-50	19

If five more people of age 21 years, 32 years, 35 years, 44 years and 11 years attend the concert, which statement describes the central tendency of the new data?

(A) The central tendency of the data increases by 0.1 as the mean increases by 0.1.

(B) The central tendency of the data increases by 0.1 as the median increases by 0.1.

(C) The central tendency of the data increases by 0.2 as the mean increases by 0.2.

(D) The central tendency of the data increases by 0.2 as the median increases by 0.2.

18. A box has 10 equal size cards. Of the 10 cards, 4 are blue, 3 are green, 2 are yellow and 1 is red. If a card is randomly drawn from the box, which is the colour that the card is most likely to have?

(A) Red (B) Blue (C) Green (D) Yellow

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. **(A) :** $\sqrt{9} - \sqrt{8}$ is an irrational number.

(R) : $\sqrt{9} - \sqrt{7} = \sqrt{2}$, which is an irrational number.

20. **(A) :** If three vertices of a parallelogram ABCD are A(3, – 4), B(–1, –3) and C(– 6, 2), then the fourth vertex D is (–2, 1).

(R) : Mid-point of AD must be equal to mid-point of BC.

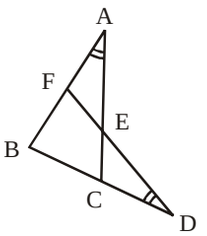
SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. For which value of k, the following pair of linear equations has no solution ?

$$2x + 3y = 1$$
$$(k - 1)\,x + (2k + 1)\,y = (k - 1)$$

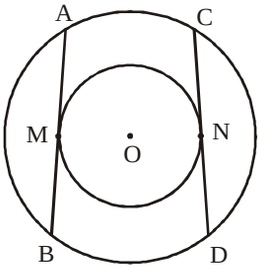
22. In the figure, if $\angle A = \angle D$, then prove that $AE \times DC = DE \times AF$.



Or

Write first four terms of the AP, when the first term is 1.25 and common difference is – 0.25.

23. In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length.



24. If $\sin(A + B) = 1$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B < 90°$ and $A > B$, then find the values of A and B.

25. How many spherical lead balls of radius 2.1 cm can be obtained from a rectangular solid lead with dimensions 88 cm, 42 cm and 21 cm ? $\left(\text{Use } \pi = \frac{22}{7}\right)$

Or

Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of the hemisphere?

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Pens are sold in packs of 8 and notepads are sold in packs of 12. Find the least number of packs of each type that one should buy so that there are equal number of pens and notepads.

27. Quadratic polynomial $4x^2 + 12x + 9$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are $\alpha - 1$ and $\beta - 1$.

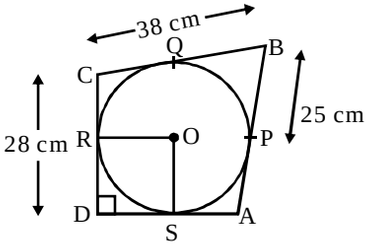
28. Find all real values of c for which the pair of linear equations $2x + y = 8$ and $5x - 2y = c$ has a unique solution.

Or

Find the values of a and b for which the following pair of linear equations has infinite number of solutions :

$$2x + 3y = 7 \text{ and } 2\alpha x + (\alpha + \beta)y = 28$$

29. In the given figure, ABCD is a quadrilateral, in which $\angle ADC = 90°$, BC = 38 cm, CD = 28 cm and BP = 25 cm. Find the radius of the circle with centre O.



30. If $\cos (A + B) = \frac{1}{2} = \sin (A - B)$, then find A and B, when it is given that A + B and A – B are acute angles.

Or

If A and B are acute angles such that :

$$\tan A = \frac{1}{3}, \tan B = \frac{1}{2} \text{ and } \tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B},$$

show that $A + B = 45°$.

31. In a leap year, the month of February begins with Monday. Find the probability that :

(*a*) its last day is also Monday.

(*b*) it has 5 Mondays.

(*c*) it has 5 Sundays.

SECTION – D

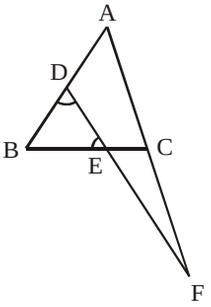
Section-D consists of 4 questions of 5 marks each.

32. A mobile company charges a fixed amount as monthly rental which includes 100 minutes free per month and charges a fixed amount thereafter for every additional minute. Abhay paid ₹ 433 for 370 minutes and Anita paid ₹ 398 for 300 minutes. Find the bill amount under the same plan, if Kareena used it for 400 minutes.

Or

The area of a rectangle reduces by 160 m^2, if its length is increased by 5 m and breadth is reduced by 4 m. However, if length is decreased by 10 m and breadth is increased by 2 m, then its area is decreased by 100 m^2. Find the dimensions of the rectangle.

33. In the figure, $\angle BED = \angle BDE$ and E is the middle point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.



34. The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of frustum is 16 cm, find its volume and total surface area.

Or

The internal radii of the ends of a bucket, full of milk and of internal height 16 cm, are 14 cm and 7 cm. If this milk is poured into a hemispherical vessel, the vessel is completely filled. Find the internal diameter of the hemispherical vessel.

35. The following frequency distribution shows the survey of heights of 50 girls and their median is given to be 151.5. Find the missing frequencies.

Height (in cm)	Number of girls
120 – 130	2
130 – 140	f_1
140 – 150	12
150 – 160	f_2
160 – 170	8

SECTION – E

Case study based questions are compulsory.

36. A manufacturer of laptop produced 6000 units in 3rd year and 7000 units in the 7th year, assuming that production increases uniformly by a fixed number every year.

Based on the above information, answer the following questions :

(*i*) Find the production in the 1st year.

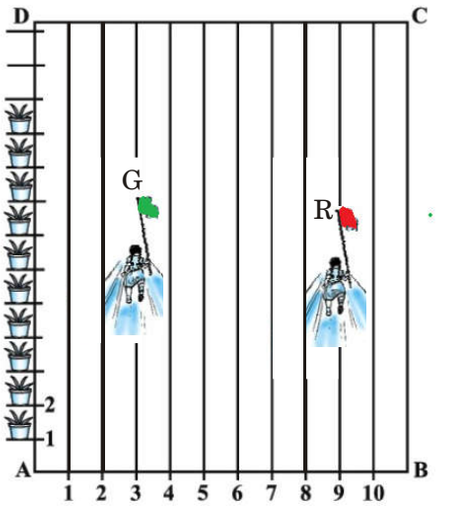
(*ii*) Find the production in the 5th year.

(*iii*) Find the production in the 6th year.

Or

By which fixed number, the production is increasing every year ?

37. In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground ABCD, 100 flowerpots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs 1/4th the distance AD on the third line and posts a green flag. Preet runs 1/5th distance AD on the ninth line and posts a red flag.



(*i*) Find the position of green flag.

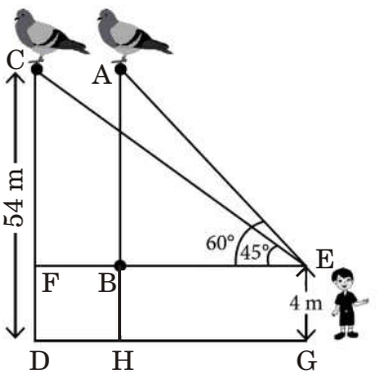
(*ii*) Find the position of red flag.

(*iii*) Find the distance between both the flags?

Or

What will be the coordinates of the point dividing the line segment joining green and red flags in the ratio 1 : 3 ?

38. A boy 4 m tall spots a pigeon sitting on the top of a pole of height 54 m from the ground. The angle of elevation of the pigeon from the eyes of boy at any instant is 60°. The pigeon flies away horizontally in such a way that it remained at a constant height from the ground. After 8 seconds, the angle of elevation of the pigeon from the same point is 45° (see figure).



Based on the above information, answer the following questions.

(*i*) Find the distance between the pole and the boy.

(*ii*) Find the distance of first position of the pigeon from the eyes of the boy.

(*iii*) How much distance the pigeon covers in 8 seconds?

Or

Find the distance of the second position of the pigeon from the eye of the boy.

5 SAMPLE QUESTION PAPER

MATHEMATICS (Standard)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions : Same as in CBSE Sample Question Paper.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. Which of the following is NOT an irrational number?

(A) $2 \times \frac{1}{\sqrt{2}}$ (B) $\sqrt{2} \times \frac{1}{2}$ (C) $\sqrt{2} \times \frac{1}{\sqrt{3}}$ (D) $\sqrt{2} \times \frac{1}{\sqrt{2}}$

2. Is $9 + \sqrt{2}$ an irrational number?

(A) No, because if $9 + \sqrt{2} = \frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b-a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

(B) No, because if $9 + \sqrt{2} = \frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b+a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

(C) Yes, because if $9 + \sqrt{2} = \frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b-a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

(D) Yes, because if $9 + \sqrt{2} = \frac{a}{b}$, where a and b are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b+a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

3. A student simplified the equation $5x^2 + 3x - 1 = 0$ as $x^2 + \frac{3}{5}x - \frac{1}{5} = 0$

Which of these could be the next step to solve the equation using the method of completing the square?

(A) $x^2 + 5(2)x + \frac{3}{5}x - \frac{1}{5} = 0$ (B) $x^2 + 5(2)x + \frac{3}{5}x - \frac{3}{5}x + \frac{1}{5} = 0$

(C) $x^2 + 2\left(\frac{3}{5}x\right) + \left(\frac{2}{3}\right)^2 - \frac{1}{3} = 0$ (D) $x^2 + \left(\frac{3}{5}\right)x + \left(\frac{3}{10}\right)^2 - \frac{9}{100} - \frac{1}{5} = 0$

4. In the equations shown below, a and b are unknown constants.

$$3ax + 4y = -2$$
$$2x + by = 14$$

If $(-3, 4)$ is the solution of the given equations, what are the values of a and b?

(A) $a = 2, b = 5$ (B) $a = -2, b = 5$ (C) $a = 5, b = 2$ (D) $a = 5, b = -2$

5. The roots of the equation $a^2x^2 - 3abx - 4b^2 = 0$ by completing the square method is

(A) $\frac{4b}{a}$ and $-\frac{b}{a}$ (B) $\frac{b}{2a}$ and $\frac{b}{a}$ (C) $\frac{2a}{b}$ and $\frac{a}{b}$ (D) $\frac{2a}{b}$ and $\frac{2b}{a}$

6. Rahul is 5 feet tall. He places a mirror on the ground and moves until he can see the top of a building. At the instant when Rahul is 2 feet from the mirror, the building is 48 feet from the mirror. How tall is the building?

(A) 120 feet (B) 96 feet (C) 240 feet (D) 180 feet

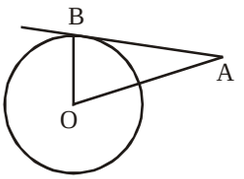
7. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then the ratio of their corresponding altitudes is :

(A) 3 : 2 (B) 16 : 81 (C) 4 : 9 (D) 2 : 3

8. A point G divides a line segment in the ratio 3:7. The segment starts at the origin and ends at a point K having 20 as its abscissa and 40 as its ordinate. Given that G is closer to the origin than to point K, which of the following are the coordinates of point G?

(A) (14, 28) (B) (28, 14) (C) (12, 6) (D) (6, 12)

9. In the given figure, $AB = \sqrt{3}\ OB$, then $\angle AOB$ is :



(A) 30° (B) 45° (C) 60° (D) 15°

10. If $\sin (A - B) = \frac{1}{2}$ and $\cos (A + B) = \frac{1}{2}$, then the value of B is :

(A) 45° (B) 60° (C) 15° (D) 0°

11. If $\sqrt{2} \sin (60° - \alpha) = 1$, then the value of α is :

(A) 45° (B) 15° (C) 60° (D) 30°

12. If $\tan (A - B) = \frac{1}{\sqrt{3}}$ and $\sin A = \frac{1}{\sqrt{2}}$, then the value of B is :

(A) 45° (B) 60° (C) 0° (D) 15°

13. Four solids spheres of the same size are made by melting a solid metallic cylinder of base diameter 4 cm and height 36 cm. What is the diameter of each sphere?

(A) 3 cm (B) 6 cm (C) 7 cm (D) 14 cm

14. Asif is melting three solid metal spheres of radii 3 cm, 4 cm, and 5 cm respectively to form a new sphere. How much more is the surface area of the new sphere as compared to the combined surface area of the three spheres?

(A) 4π (B) 16π (C) 36π (D) 64π

15. A cube of side length 66 cm is filled with spherical metallic balls of diameter 0.6 cm and it is assumed that $\frac{5}{8}$ space of the cube remains unfilled. What are number of balls that can be filled in the cube? $\left(\text{Use } \pi = \frac{22}{7}\right)$

(A) 952875 (B) 959777 (C) 1165812 (D) 2264031

16. The lower limit of the modal class of the following data is :

Class interval	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	5	15	6	10	14	9

(A) 25 (B) 15 (C) 10 (D) 30

17. The lower limit of the median class of the following data is :

C.I.	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	8	13	7	6

(A) 10 (B) 30 (C) 20 (D) 50

18. Of 50 students in a class, 16 prefer cricket, 8 prefer football, 7 prefer basketball and rest of the students prefer either tennis or hockey. There are twice as many students who prefer tennis as the number of students who prefer hockey. A student is randomly selected from the class. Which statement is correct?

(A) The probability of selecting a student who prefer tennis is more than that of selecting a student who prefer cricket.

(B) The probability of selecting a student who prefer hockey is more than that of selecting a student who prefer cricket.

(C) The probability of selecting a student who prefer hockey is more than that of selecting a student who prefer tennis.

(D) The probability of selecting a student who prefer basketball is more than that of selecting a student who prefer cricket.

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. **(A) :** Product of $(2+\sqrt{3})$ and $(3+\sqrt{5})$ is an irrational number.

(R) : Product of two irrational numbers is an irrational number.

20. **(A) :** A(1, 7), B(4, 2), C(–1, –1) and D(–4, 4) are vertices of a rhombus.

(R) : Here, AB = BC = CD.

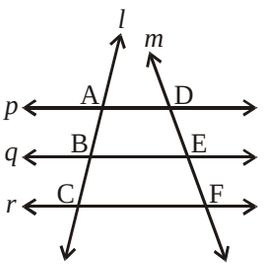
SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. For what value of k, does the pair of equations given below has a unique solution ?

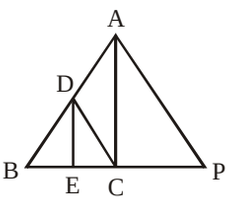
$$y - x = 6$$
$$3kx + 2y = 7$$

22. Three parallel lines p, q and r are intersected by two transversals l and m at A, B, C and D, E, F respectively as shown in the figure. Prove that $\frac{AB}{BC} = \frac{DE}{EF}$.

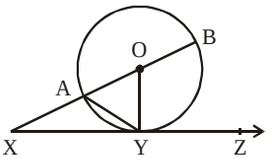


Or

In the figure of ΔABC, DE || AC. If DC || AP, where point P lies on BC produced, then prove that $\frac{BE}{EC} = \frac{BC}{CP}$.



23. In the figure, tangent XZ touches the circle with centre O at Y. Diameter BA when produced meets XZ at X. If $\angle BXY = b$ and $\angle AYX = a$, prove that $b + 2a = 90°$.



24. Write the values of cos 0°, cos 30°, cos 45°, cos 60° and cos 90°. What happens to the values of cos θ as θ increases from 0° to 90° ?

25. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder of the same radius. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Or

Two cubes of side 5 cm each are kept together joining edge to edge to form a cuboid. Find the surface area of the cuboid so formed.

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

27. If one zero of the polynomial $2x^2 - 5x - (2k + 1)$ is twice the other, find both the zeroes of the polynomial and the value of k.

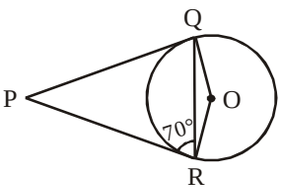
28. The sum of the digits of a two digit number is 9. If, nine times this number is equal to twice the number obtained by reversing the order of digits of the number, find the number.

Or

Solve the following pair of equations :

$$49x + 51y = 499$$
$$51x + 49y = 501$$

29. In the figure, PQ and PR are tangents drawn to a circle with centre O from an external point P. If $\angle PRQ = 70°$, then find $\angle QPR$ and $\angle OQR$.



30. In ΔABC, right angled at C, if $\tan A = \frac{1}{\sqrt{3}}$, show that $\sin A.\cos B + \cos A. \sin B = 1$.

Or

Find the value of tan 30° geometrically.

31. Three unbiased coins are tossed together. What is the probability of getting (*a*) two heads, (*b*) at least two heads, (*c*) at most two heads ?

SECTION – D

Section-D consists of 4 questions of 5 marks each.

32. Draw the graph of the following pair of linear equations :

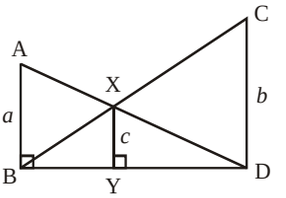
$$x + y = 8$$
$$6x - 5y = 15$$

Shade the region bounded by these lines and $x = 0$. Also, write the coordinates of the vertices of the triangle.

Or

For what value of x both the polynomials $x^2 - 3x + 2$ and $x^2 - 6x + 5$ become zero ?

33. In the figure, if $\angle ABD = \angle XYD = \angle CDB = 90°$, AB = a, XY = c and CD = b, then prove that $c\,(a + b) = ab$.



34. A solid metallic right circular cone of diameter 14 cm and height 8 cm is melted and recast into a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere.

Or

An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin required to make the funnel.

35. In a check up of heart beats rate of 50 females, it was found that median heart beats is 78. Find the missing frequencies f_1 and f_2 in the following frequency distribution :

Number of heart beats per minute	Number of females
64 – 68	4
68 – 72	5
72 – 76	f_1
76 – 80	f_2
80 – 84	9
84 – 88	7
88 – 92	1

SECTION – E

Case study based questions are compulsory.

36. In Mathematics, relations can be expressed in various ways. The matchstick patterns are based on linear relations. Different strategies can be used to calculate the number of matchsticks used in different figures.

One such pattern is shown below. Observe the pattern and answer the following questions using Arithmetic Progression :

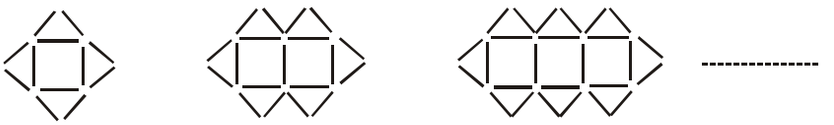


Figure 1 *Figure 2* *Figure 3* ----------

(*i*) Write the AP for the number of triangles used in the figures.

(*ii*) Write the AP for the number of matchsticks used in the above figures.

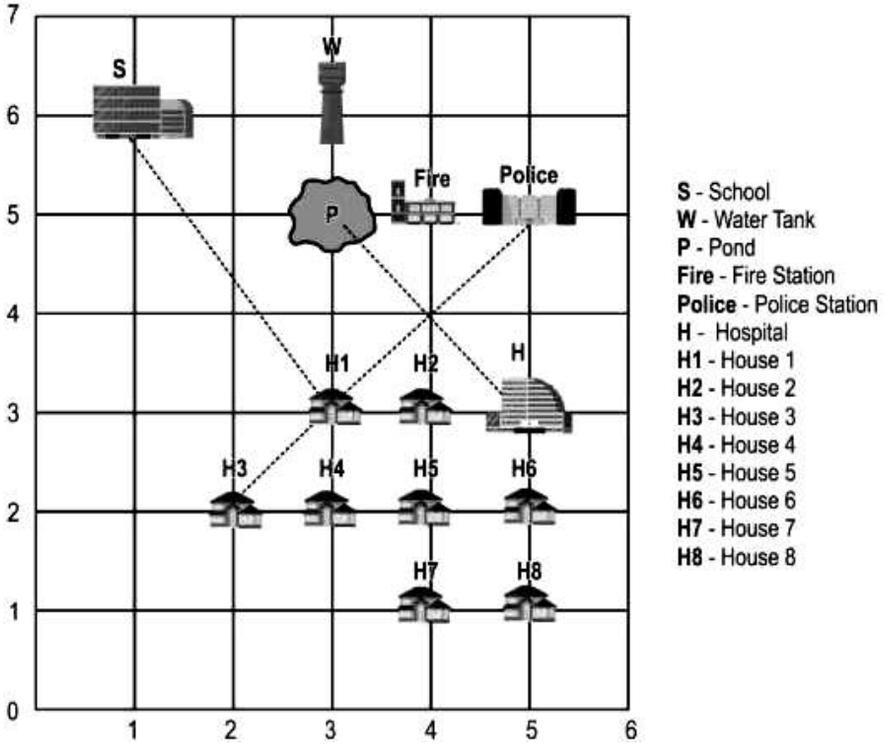
(*iii*) Write the *n*th term of this AP.

Or

Which figures has 61 matchstickes?

37. Study the given information and answer the questions that follow.

Shown below is a town plan on a coordinate grid, where 1 unit = 1 km. Consider the co-ordinates of each building to be the point of intersection of the respective grid lines.



(*Note :* Consider the horizontal axis as the *x*-axis and the vertical axis as the *y*-axis.)

(*i*) Write the coordinates of House H1.

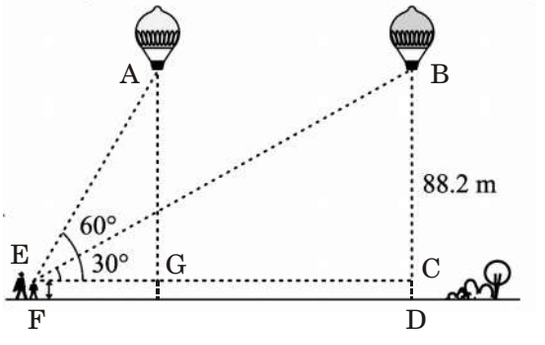
(*ii*) Write the coordinates of Police Station.

(*iii*) Find the ratio in which House 1 divides the path joining House 3 and Police Station.

Or

A well is dug at a point along the path joining the pond and the hospital. The ratio of the distance between the pond and the well to that of the well and the hospital is 4:1 respectively. What is the *x*-coordinate of the well?

38. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After 30 seconds, the angle of elevation reduces to 30° (see the figure given alongside).



Based on the above information, answer the following questions.

(*i*) Find the distance of first position of balloon from the eye of the girl.

(*ii*) Find the distance of second position of balloon from the eye of the girl.

(*iii*) Find the distance travelled by the balloon during the above interval.

Or

Find the speed of the balloon.

6 SAMPLE QUESTION PAPER

MATHEMATICS (Standard)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions : Same as in CBSE Sample Question Paper.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. $(2+\sqrt{5})(2+\sqrt{5})$ is :

(A) A rational number (B) A whole number

(C) An irrational number (D) A natural number

2. $(\sqrt{2}-\sqrt{3})(\sqrt{3}+\sqrt{2})$ is :

(A) A rational number (B) A whole number

(C) An irrational number (D) A natural number

3. Which is the correct way to verify that 2 and 3 are the roots of the equation $x^2 - 5x + 6 = 0$?

(A) On substituting $x = 2$ and $x = 3$ on the left-hand side of the equation, the result should be 0

(B) On substituting x^2 with 2 and x with 3 on the left-hand side of the equation, the result should be 0.

(C) On substituting $x = 2$ on the left-hand side of the equation, the result should be 3.

(D) On substituting $x = 3$ on the left-hand side of the equation, the result should be 2.

4. Consider the equations shown :

$$ax + by = ab \text{ and } 2ax + 3by = 3b$$

Which of these is the value of y in terms of a?

(A) $y = 2a - 35$ (B) $y = 2ab - 3b$ (C) $y = 9a - 35$ (D) $y = 3 - 2a$

5. A student solved a quadratic equation and obtains the roots as -4 and 3. Part of the student's work to verify the root is shown :

$$(-4)^2 + 2(-4) - 9 = 0.$$

Based on the student's work, which of these is correct?

(A) The student calculated the roots correctly.

(B) The student calculated the roots correctly but should replace $2(-4)$ in his work with $2(3)$.

(C) The student calculated the roots of the equation that can be obtained by adding 1 to the equation that the student solved.

(D) The student calculated the roots of the equation that can be obtained by adding -1 to the equation that the student solved.

6. If $\Delta ABC \sim \Delta DEF$ such that $\angle A = 45°$ and $\angle E = 56°$, then the value of $\angle C$ is

(A) 45° (B) 79° (C) 56° (D) 90°

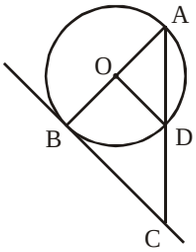
7. In ΔABC and ΔDEF, if $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$, then ΔABC and ΔDEF are similar. Symbolically we write :

(A) $\Delta ABC = \Delta DEF$ (B) $\Delta ABC \cong \Delta DEF$ (C) $\Delta BAC \sim \Delta DEF$ (D) $\Delta ABC \sim \Delta DEF$

8. On a coordinate grid, the location of a bank is $(-4, 8)$ and the location of a post office is $(2, 0)$. The scale used is 1 unit = 50 m. What is the shortest possible distance between the bank and the post office?

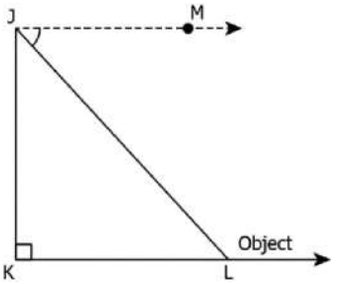
(A) 200 m (B) 500 m (C) 700 m (D) 800 m

9. In the figure, BC is a tangent, O is the centre of the circle and $\angle AOD = 80°$, then $\angle BCA$ is :



(A) 50° (B) 40° (C) 80° (D) 90°

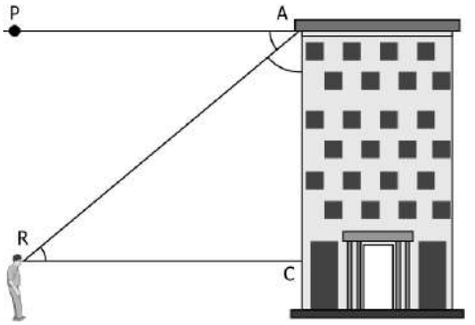
10. Consider the figure shown.



If $\angle LJM$ is the angle of depression, what is the line of sight in the figure shown?

(A) JM (B) JL (C) JK (D) KL

11. Observe the figure below.



If AR is the line of sight, which of these angles represents the angle of elevation?

(A) $\angle ARC$ (B) $\angle RAP$ (C) $\angle ACR$ (D) $\angle RAC$

12. The angles of elevation of two cars, from the car to the top of a building are 45° and 30°. If the cars are on the same side of the building and are 50 m apart, what is the height of the building?

(A) $25(\sqrt{3} - 1)$ m (B) $25(\sqrt{3} + 1)$ m (C) $50(\sqrt{3} - 1)$ m (D) $50(\sqrt{3} + 1)$ m

13. If a metallic cube of edge 1 cm is drawn into a wire of diameter 4 mm, then the length of the wire is :

(A) $100/\pi$ cm (B) $100\ \pi$ cm (C) $25/\pi$ cm (D) 10000 cm

14. A metallic cube of edge 2 cm is drawn into a wire of diameter 4 mm, then, the length of the wire is

(A) $\frac{800}{\pi}$ cm (B) 800 cm (C) $\frac{200}{\pi}$ cm (D) 80000 cm

15. The radius of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is :

(A) 4.2 cm (B) 8.4 cm (C) 1.05 cm (D) 2.1 cm

16. According to an Empirical formula, which is correct ?

(A) Mean > Mode > Median (B) Mean < Mode < Median

(C) Mode = $\frac{\text{Mean} + \text{Median}}{2}$ (D) Mean = Median = Mode

17. Mode and mean of a data is 12.6 and 10.5 respectively. Its median is

(A) 35.7 (B) 11.9 (C) 33.6 (D) 11.2

18. A card is drawn from a well shuffled pack of cards. What is the chance that it is a red face card ?

(A) $\frac{5}{26}$ (B) $\frac{3}{13}$ (C) $\frac{3}{26}$ (D) $\frac{11}{36}$

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. **(A) :** Product of 6 and $\sqrt{7}$ is an irrational number.

(R) : Product of a non-zero rational number and an irrational number is an irrational number.

20. **(A) :** If a point P(x, y) is equidistant from the points A(–1, 5) and B(5, 1), then the relation between x and y is $2x - 3y = 0$.

(R) : $PA^2 = PB^2$ Þ $(x + 1)^2 + (y - 5)^2 = (x - 5)^2 + (y - 1)^2$.

SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. Find whether the following pair of linear equations is consistent or inconsistent :

$$y - x = 1$$
$$x - 3y = 4$$

22. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle is bisected at the point of contact.

Or

Two concentric circles with centre O are of radii 5 cm and 3 cm. From an external point P, two tangents PA and PB are drawn to these circles respectively. If PA = 12 cm, then find length of PB.

23. A circle with centre O, touches the sides of a quadrilateral ABCD at P, Q, R and S. Prove that $\angle AOB + \angle COD = 180°$.

24. If $\sin(A + B) = 1$ and $\sin(A - B) = \frac{1}{2}$; $0 \le A + B \le 90°$ and $A > B$, then find A and B.

25. Three cubes of a metal, whose edges are in the ratio 3 : 4 : 5, are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

Or

Some children, playing on the beach, dig out sand for making a hollow cylinder in the ground of radius 14 cm and depth 20 cm. They then use this sand to make a cone like structure of radius 14 cm. What is the height of the cone ?

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

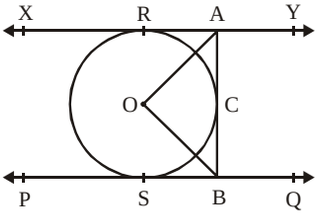
27. Find the zeroes of the quadratic polynomial $t^2 + 8t + 16$ and verify the relationship between the zeroes and the coefficients.

28. A man earns ₹ 600 per month more than his wife. One-tenth of the man's salary and one-sixth of the wife's salary amount to ₹ 1500, which is saved every month. Find their incomes.

Or

Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

29. If XY and PQ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C is intersecting XY at A and PQ at B, prove that $\angle AOB = 90°$.



30. Prove that : $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} - \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{4}{\cot\theta - \tan\theta} = \frac{4\tan\theta}{1 - \tan^2\theta}$.

Or

Prove that : $\tan\theta - \cot\theta = \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta} = \frac{1 - 2\cos^2\theta}{\sin\theta\cos\theta}$.

31. A bingo caller has machine that contains 65 balls. The balls are marked as follows :

B1, B2, B3, ..., B13

P1, P2, P3, ..., P13

N1, N2, N3, ..., N13

G1, G2, G3, ..., G13

O1, O2, O3, ..., O13

The balls are mixed and one ball is drawn at random. Find

(*i*) P (drawing a ball that has G on it)

(*ii*) P (drawing a ball that has a number which is not a multiple of 5)

(*iii*) P (drawing a ball that has a doublet *i.e.* 11)

SECTION – D

Section-D consists of 4 questions of 5 marks each.

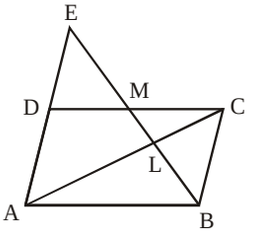
32. For what values of m and n the following system of linear equations has infinitely many solutions ?

$$3x + 4y = 12$$
$$(m + n)x + 2(m - n)y = 5m - 1$$

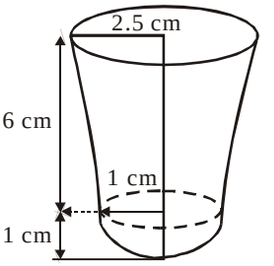
Or

Find an AP whose sum of the first three terms is 21 and the sum of their squares is 155.

33. In a parallelogram ABCD, middle point of CD is M. A line segment BM is drawn which cuts AC at L and meets AD extended at E. Prove that EL = 2BL.



34. A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere (see figure). The diameters of the ends of the frustum are 5 cm and 2 cm and the height of the entire shuttle cock is 7 cm. Find the external surface area of the shuttle-cock. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Or

A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts in the middle of its height by a plane parallel to the base. If the frustum so obtained is drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

35. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	Frequency
0 – 10	5
10 – 20	x
20 – 30	20
30 – 40	15
40 – 50	y
50 – 60	5
Total	60

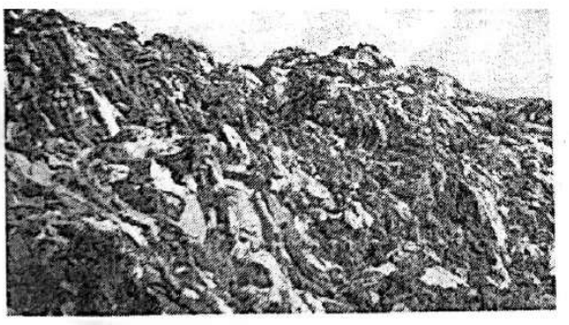
SECTION – E

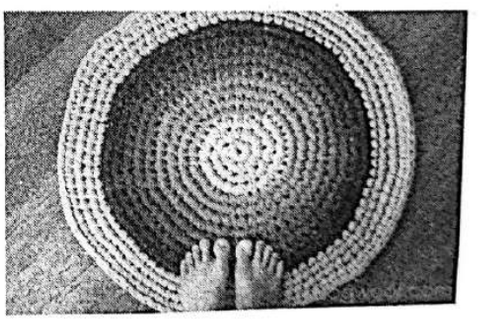
Case study based questions are compulsory.

36. Do you know old clothes which are thrown as waste not only fill the landfill site but also produce very harmful greenhouse gas. So, it is very important that we reuse old clothes in whatever way we can.

The picture given below on the right, shows a footmat (rug) made out of old t-short yarn. Observing the picture, you will notice that a number of stitches in circular rows are making a pattern :

6, 12, 18, 24, ...





Based on the above information, answer the following questions :

(*i*) Check whether the given pattern forms an AP. If yes, find the common difference and the next term of the AP.

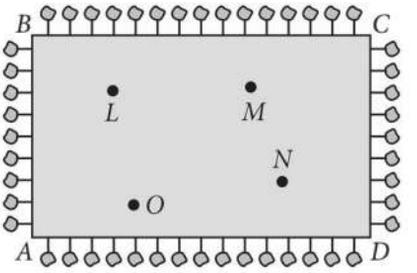
(*ii*) Find the number of stitches in the 10th circular row.

(*iii*) Write the *n*th term of the AP.

Or

Find the total number of stitches upto the 10th circular row ?

37. On the occasions of 'Diwali' a rectangular plot have been allotted for 'Diwali Mela' to students of secondary school in Hyderabad. In order to reduce smog and pollution they decided to keep little leaf linden plant on the boundary at a distance of 1 m from each other. Four air purifier machines have also been set up at points L, M, N, O. (Answer the following questions considering A as origin).



(*i*) What are the coordinates of L?

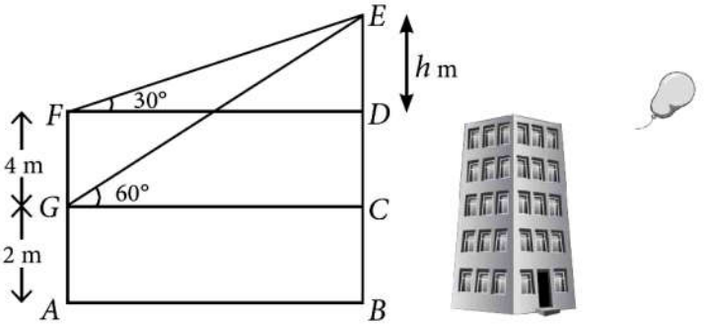
(*ii*) What are the coordinates of N?

(*iii*) Find the distance between L and O.

Or

Find the mid-point of the segment joining the points L and N.

38. There are two windows in a house. First window is at the height of 2 m above the ground and other window is 4 m vertically above the lower window. Ankit and Radha are sitting inside the two windows at points G and F respectively. At an instant, the angles of elevation of a balloon from these windows are observed to be 60° and 30° as shown below.



Based on the above information, answer the following questions :

(*i*) Find the value of *h*.

(*ii*) What is the horizontal distance of the balloon from the house?

(*iii*) What is the height of the balloon from the ground?

Or

Find the distance of the balloon from the window G.

YEAR 2022-23

CBSE SAMPLE QUESTION PAPER

MATHEMATICS (Basic)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions :

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 2 marks, 2 Questions of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is

(a) ab (b) a^2b^2 (c) a^3b^2 (d) a^3b^3

Ans. (c) a^3b^2 1

2. What is the greatest possible speed at which a man can walk 52 km and 91 km in an exact number of hours?

(a) 17 km/hour (b) 7 km/hour (c) 13 km/hour (d) 26 km/hour

Ans. (c) 13 km/hour 1

3. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) -10 (c) 5 (d) -5

Ans. (b) -10 1

4. Graphically, the pair of equations given by

$$6x - 3y + 10 = 0$$
$$2x - y + 9 = 0$$

represents two lines which are

(a) intersecting at exactly one point. (b) parallel.

(c) coincident. (d) intersecting at exactly two points.

Ans. (b) parallel. 1

5. If the quadratic equation $x^2 + 4x + k = 0$ has real and equal roots, then

(a) $k < 4$ (b) $k > 4$ (c) $k = 4$ (d) $k \geq 4$

Ans. (c) $k = 4$ 1

6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(a) 5 units (b) 12 units (c) 11 units (d) $(7 + \sqrt{5})$ units

Ans. (b) 12 units 1

7. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(*a*) $\angle B = \angle E$ (*b*) $\angle A = \angle D$ (*c*) $\angle B = \angle D$ (*d*) $\angle A = \angle F$

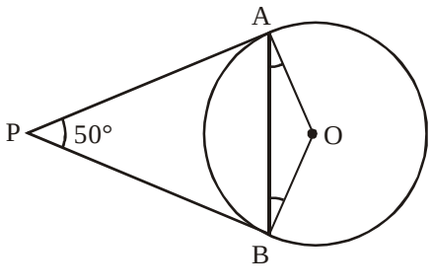
Ans. (*c*) $\angle B = \angle D$ 1

8. In which ratio the *y*-axis divides the line segment joining the points (5, – 6) and (–1, – 4) ?

(*a*) 1 : 5 (*b*) 5 : 1 (*c*) 1 : 1 (*d*) 1 : 2

Ans. (*b*) 5 : 1 1

9. In the figure, if PA and PB are tangents to a circle with centre O such that $\angle APB = 50°$, then $\angle OAB$ is equal to



(*a*) 25° (*b*) 30° (*c*) 40° (*d*) 50°

Ans. (*a*) 25° 1

10. If $\sin A = \frac{1}{2}$, then the value of cos A is :

(*a*) $\frac{\sqrt{3}}{2}$ (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\sqrt{3}$ (*d*) 1

Ans. (*a*) $\frac{\sqrt{3}}{2}$ 1

11. $\sqrt{3}\ \cos^2 A + \sqrt{3}\ \sin^2 A$ is equal to

(*a*) 1 (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\sqrt{3}$ (*d*) 0

Ans. (*c*) $\sqrt{3}$ 1

12. The value of cos 1° cos 2° cos 3° cos 4° ..... cos 90° is

(*a*) 1 (*b*) 0 (*c*) –1 (*d*) 2

Ans. (*b*) 0 1

13. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(*a*) 22 : 7 (*b*) 14 : 11 (*c*) 7 : 22 (*d*) 11: 14

Ans. (*b*) 14 : 11 1

14. If the radii of two circles are in the ratio of 4 : 3, then their areas are in the ratio of :

(*a*) 4 : 3 (*b*) 8 : 3 (*c*) 16 : 9 (*d*) 9 : 16

Ans. (*c*) 16 : 9 1

15. The total surface area of a solid hemisphere of radius 7 cm is :

(*a*) 447π cm^2 (*b*) 239π cm^2 (*c*) 174π cm^2 (*d*) 147π cm^2

Ans. (*d*) 147π cm^2 1

16. For the following distribution :

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

the upper limit of the modal class is

(*a*) 10 (*b*) 15 (*c*) 20 (*d*) 25

Ans. (*c*) 20 1

17. If the mean of the following distribution is 2.6, then the value of y is

Variable (x)	1	2	3	4	5
Frequency	4	5	y	1	2

(a) 3 (b) 8 (c) 13 (d) 24

Ans. (b) 8 1

18. A card is selected at random from a well shuffled deck of 52 cards. The probability of its being a red face card is

(a) $\frac{3}{26}$ (b) $\frac{3}{13}$ (c) $\frac{2}{13}$ (d) $\frac{1}{2}$

Ans. (a) $\frac{3}{26}$ 1

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

19. Assertion : If HCF of 510 and 92 is 2, then the LCM of 510 and 92 is 32460.
Reason : As HCF (a, b) × LCM (a, b) = a × b

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Ans. (d) Assertion (A) is false but Reason (R) is true. 1

20. Assertion (A) : The ratio in which the line segment joining (2, – 3) and (5, 6) internally divided by x-axis is 1 : 2.

Reason (R) : As formula for the internal division is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Ans. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). 1

SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. 21 For what values of k will the following pair of linear equations have infinitely many solutions?

$$kx + 3y - (k - 3) = 0$$
$$12x + ky - k = 0$$

Sol. For a pair of linear equations to have infinitely many solutions :

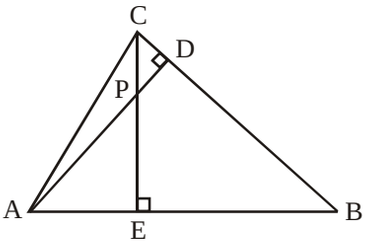
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$$ ½

$$\frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$$ ½

Also, $\frac{3}{k} = \frac{k-3}{k} \Rightarrow \quad k^2 - 6k = 0 \Rightarrow \quad k = 0, 6.$ ½

Therefore, the value of k, that satisfies both the conditions, is k = 6. ½

22. In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that :



(*i*) ΔABD ~ ΔCBE (*ii*) ΔPDC ~ ΔBEC

Sol. (*i*) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB = 90°$ ½

$\angle ABD = \angle CBE$ (Common angle)

$\Rightarrow$ ΔABD ~ ΔCBE (AA criterion) ½

(*ii*) In ΔPDC and ΔBEC,

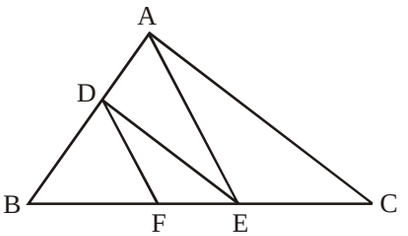
$\angle PDC = \angle BEC = 90°$ ½

$\angle PCD = \angle BCE$ (Common angle)

$\Rightarrow$ ΔPDC ~ ΔBEC (AA criterion) ½

Or

In the figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Sol. In ΔABC, DE || AC,

$\frac{BD}{AD} = \frac{BE}{EC}$ ...(*i*) (Using BPT) ½

In ΔABE, DF || AE

$\frac{BD}{AD} = \frac{BF}{FE}$ ...(*ii*) (Using BPT) ½

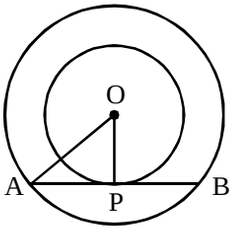
From (*i*) and (*ii*),

$\frac{BD}{AD} = \frac{BE}{EC} = \frac{BF}{FE}$ ½

Thus, $\frac{BF}{FE} = \frac{BE}{EC}$. ½

23. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Sol. Let O be the centre of the concentric circle of radii 5 cm and 3 cm respectively. Let AB be a chord of the larger circle touching the smaller circle at P.



Then AP = PB and OP ⊥ AB.

Applying Pythagoras theorem in ΔOPA, we have ½

$OA^2 = OP^2 + AP^2 \Rightarrow 25 = 9 + AP^2$ ½

$\Rightarrow$ $AP^2 = 16 \Rightarrow AP = 4$ cm ½

$\therefore$ $AB = 2AP = 8$ cm ½

24. If $\cot\theta = \frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

Sol. Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$ ½

$= \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$ ½

$= \cot^2\theta$ ½

$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$ ½

25. Find the perimeter of a quadrant of a circle of radius 14 cm.

Sol. Perimeter of the quadrant $= 2r + \frac{1}{4} \times 2\pi r$ ½

$\Rightarrow$ Perimeter $= 2 \times 14 + \frac{1}{2} \times \frac{22}{7} \times 14$ ½

$\Rightarrow$ Perimeter $= 28 + 22 = 50$ cm 1

Or

Find the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm.

Sol. Area of the given circle = Area of first circle + Area of second circle

$\Rightarrow$ $\pi r^2 = \pi(r_1)^2 + \pi(r_2)^2$ ½

$\Rightarrow$ $\pi r^2 = \pi(24)^2 + \pi(7)^2 \Rightarrow \pi r^2 = 576\pi + 49\pi = 625\pi$ ½

$\Rightarrow$ $r^2 = 625 \Rightarrow r = 25$ cm

Thus, diameter of the circle $= 2r = 2 \times 25 = 50$ cm. 1

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{5}$ is an irrational number.

Sol. Let us assume to the contrary, that $\sqrt{5}$ is rational. Then, we can find integers a and b ($\neq 0$) such that $\sqrt{5} = \frac{a}{b}$ (assuming that a and b are co-prime). 1

So, $a = \sqrt{5}\,b \Rightarrow a^2 = 5b^2$ ...(1)

Here 5 is a prime number that divides a^2. So, 5 divides a also.

(Using the theorem, if a is a prime number and if a divides p^2, then a divides p, where a is a positive integer.) ½

Thus, 5 is a factor of a.

Since 5 is a factor of a, we can write $a = 5c$ (where c is a positive integer). Substituting $a = 5c$ in (1), we get $(5c)^2 = 5b^2 \Rightarrow 5c^2 = b^2$ ½

This means 5 divides b^2. So, 5 divides b also (Using the theorem, if a is a prime number and if a divides p^2, then a divides p, where a is a positive integer).

Hence a and b have at least 5 as a common factor. ½

But this contradicts the fact that a and b are co-prime. This is the contradiction to our assumption that p and q are co-prime.

So, $\sqrt{5}$ is not a rational number. Therefore, the $\sqrt{5}$ is irrational. ½

27. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

Sol. $6x^2 - 7x - 3 = 0 \Rightarrow 6x^2 - 9x + 2x - 3 = 0$

$\Rightarrow 3x(2x - 3) + 1(2x - 3) = 0 \Rightarrow (2x - 3)(3x + 1) = 0$ ½

$\Rightarrow 2x - 3 = 0$ and $3x + 1 = 0$

$x = \frac{3}{2}$ and $x = \frac{-1}{3}$

Hence, the zeroes of the quadratic polynomial are $\frac{3}{2}$ and $\frac{-1}{3}$. ½

For verification,

Sum of zeroes = $\frac{\text{coefficient of } x}{\text{coefficient of } x^2} \Rightarrow \frac{3}{2} + \left(\frac{-1}{3}\right) = -\frac{(-7)}{6} \Rightarrow \frac{7}{6} = \frac{7}{6}$ 1

Product of zeroes = $\frac{\text{constant}}{\text{coefficient of } x^2} \Rightarrow \frac{3}{2} \times \left(\frac{-1}{3}\right) = -\frac{(-3)}{6} \Rightarrow \frac{-1}{2} = \frac{-1}{2}$ 1

Therefore, the relationship between zeroes and their coefficients is verified.

28. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for six days, while Anand paid ₹ 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

Sol. Let the fixed charge be ₹ x and additional charge be ₹ y per day.

Number of days for Latika = 6 = 2 + 4

Hence, charge $x + 4y = 22$

$\Rightarrow x = 22 - 4y$ ...(1) ½

Number of days for Anand = 4 = 2 + 2

Hence, charge $x + 2y = 16$

$\Rightarrow x = 16 - 2y$ ...(2) ½

On comparing equations (1) and (2), we get

$22 - 4y = 16 - 2y \Rightarrow 2y = 6 \Rightarrow y = 3$

Substituting $y = 3$ in equation (1), we get,

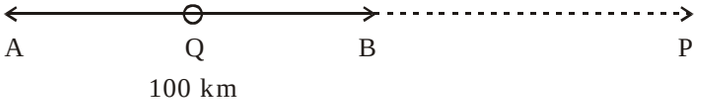
$x = 22 - 4(3) \Rightarrow x = 22 - 12 \Rightarrow x = 10$ 1

Therefore, fixed charge = ₹ 10 and additional charge = ₹ 3 per day.

Or

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Sol.



AB = 100 km. We know that, Distance = Speed × Time. ½

If the cars travel in the same direction, let they meet at P. If the cars travel towards each other, let they meet at Q.

AP – BP = 100 $\Rightarrow$ $5x - 5y = 100 \Rightarrow x - y = 20$ ...(i) ½

AQ + BQ = 100 $\Rightarrow$ $x + y = 100$ ...(ii)

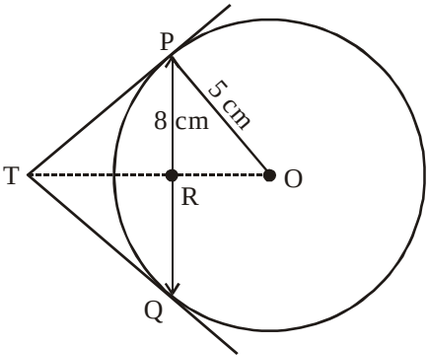
Adding equations (i) and (ii), we get,

$x - y + x + y = 20 + 100 \Rightarrow 2x = 120 \Rightarrow x = 60$ 1

Substituting $x = 60$ in equation (ii), we get, $60 + y = 100 \Rightarrow y = 40$.

Therefore, the speed of the first car is 60 km/hr and the speed of the second car is 40 km/hr. 1

29. In the figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.



Sol. Since OT is perpendicular bisector of PQ.

Therefore, $PR = RQ = 4$ cm ½

Now, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2} = 3$ cm ½

Now, $\angle TPR + \angle RPO = 90°$ $(\because$ TPO = 90°)

and $\angle TPR + \angle PTR = 90°$ $(\because$ TRP = 90°) ½

So, $\angle RPO = \angle PTR$ ½

So, $\Delta TRP \sim \Delta PRO$ [By A-A rule of similar triangles]

So, $\frac{TP}{PO} = \frac{RP}{RO}$ ½

$\Rightarrow \frac{TP}{5} = \frac{4}{3} \Rightarrow TP = \frac{20}{3}$ cm ½

30. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$.

Sol. LHS $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$ ½

$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta(1-\tan\theta)}$ ½

$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)} = \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$

$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$ ½

$= \frac{(\tan^2\theta + \tan\theta + 1)}{\tan\theta}$

$= \tan\theta + 1 + \cot\theta = 1 + \tan\theta + \cot\theta$ ½

$= 1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= 1 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}$ ½

$= 1 + \frac{1}{\sin\theta.\cos\theta} = 1 + \sec\theta.\operatorname{cosec}\theta$ ½

Or

If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Sol. $\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$ ½

$\Rightarrow \sin^2\theta + \cos\theta^2 + 2\sin\theta.\cos\theta = 3$ ½

$\Rightarrow 1 + 2\sin\theta.\cos\theta = 3 \Rightarrow 2\sin\cos\theta = 2$

$\Rightarrow \sin\theta.\cos\theta = 1$ ½

Now, $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ ½

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ ½

$= \dfrac{1}{\sin\theta\cos\theta} = \dfrac{1}{1} = 1$ ½

31. Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is
(*i*) 8? (*ii*) 13? (*iii*) less than or equal to 12?

Sol. (*i*) $P(8) = \dfrac{5}{36}$ (Favourable 2, 6; 6, 2, 3, 5; 5, 3; 4, 4) 1

(*ii*) $P(13) = \dfrac{0}{36} = 0$ 1

(*iii*) P(less than or equal to 12) = 1 1

SECTION – D

Section-D consists of 4 questions of 5 marks each.

32. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Sol. Let the average speed of passenger train = x km/h.
and the average speed of express train = $(x + 11)$ km/h. ½

As per given data, time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance. Therefore,

$\dfrac{132}{x} - \dfrac{132}{x+11} = 1$ 1

$\Rightarrow \dfrac{132(x + 11 - x)}{x(x+11)} = 1 \Rightarrow \dfrac{132 \times 11}{x(x+11)} = 1$ ½

$\Rightarrow 132 \times 11 = x(x + 11) \Rightarrow x^2 + 11x - 1452 = 0$

$\Rightarrow x^2 + 44x - 33x - 1452 = 0$ 1

$\Rightarrow x(x + 44) - 33(x + 44) = 0 \Rightarrow (x + 44)(x - 33) = 0$ 1

$\Rightarrow x = -44, 33$ ½

As the speed cannot be negative, the speed of the passenger train will be 33 km/h and the speed of the express train will be 33 + 11 = 44 km/h. ½

Or

A motor boat whose speed in still water is 18 km/h takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol. Let the speed of the stream be x km/hr. ½

So, the speed of the boat in upstream = $(18 - x)$ km/hr
and the speed of the boat in downstream = $(18 + x)$ km/hr ½

According to the question,

$$\frac{\text{Distance}}{\text{Upstream speed}} - \frac{\text{Distance}}{\text{Downstream speed}} = 1$$

$\Rightarrow \quad \frac{24}{18 - x} - \frac{24}{18 + x} = 1$ 1

$\Rightarrow \quad 24\left[\frac{1}{18 - x} - \frac{1}{18 + x}\right] = 1 \quad \Rightarrow \quad 24\left[\frac{18 + x - (18 - x)}{(18 - x).(18 + x)}\right] = 1$ 1

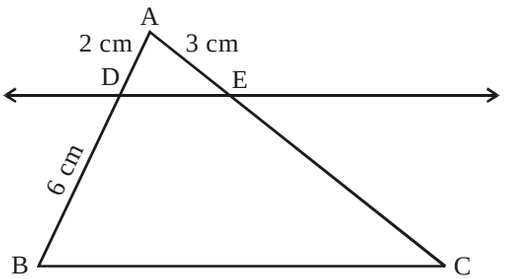
$\Rightarrow \quad 24\left[\frac{2x}{(18 - x).(18 + x)}\right] = 1$

$\Rightarrow \quad 48x = 324 - x^2 \quad \Rightarrow \quad x^2 + 48x - 324 = 0$ 1

$\Rightarrow \quad (x + 54)(x - 6) = 0 \quad \Rightarrow \quad x = -54 \text{ or } 6$ ½

As speed to stream can never be negative, the speed of the stream is 6 km/hr. ½

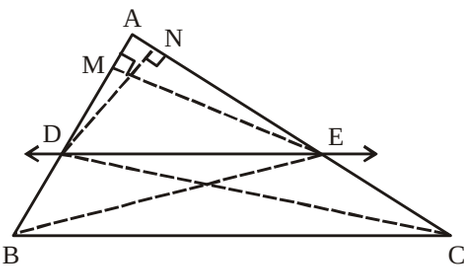
33. Prove that If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. In the figure, find EC if $\frac{\mathbf{AD}}{\mathbf{DB}} = \frac{\mathbf{AE}}{\mathbf{EC}}$ using the above theorem.



Sol. Given : A ΔABC in which $DE \parallel BC$, which intersects AB in D and AC in E. ½

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$ 1

Construction : Join BE and CD, draw $EM \perp AB$ and $DN \perp AC$.



½

Proof : We have :

$$\frac{\text{ar }(\Delta ADE)}{\text{ar }(\Delta DBE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB} \quad \ldots(1)$$

Similarly,

$$\frac{\text{ar }(\Delta ADE)}{\text{ar }(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC} \quad \ldots(2)$$

Since ΔDBE and ΔCDE stand on the same base DE and lie between the same parallels DE and BC, so

$$\text{ar }(\Delta DBE) = \text{ar }(\Delta CDE) \quad \ldots(3)$$

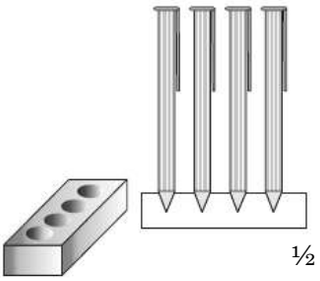
From (1), (2) and (3), we have :

$$\frac{\text{ar }(\Delta ADE)}{\text{ar }(\Delta DBE)} = \frac{\text{ar }(\Delta ADE)}{\text{ar }(\Delta CDE)}$$

$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$ [From (1) and (2)] 2

Application $\frac{2}{6} = \frac{3}{EC} \Rightarrow EC = \frac{6 \times 3}{2} = 9$ cm. 1

34. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.



Sol. Volume of one conical depression $= \frac{1}{3} \times \pi r^2 h$ ½

$= \frac{1}{3} \times \frac{22}{7} \times 0.5^2 \times 1.4 \text{ cm}^3 = 0.366 \text{ cm}^3$ 1½

Volume of 4 conical depression $= 4 \times 0.366 \text{ cm}^3$

$= 1.464 \text{ cm}^3$ ½

Volume of cuboidal box $= L \times B \times H$ ½

$= 15 \times 10 \times 3.5 \text{ cm}^3$

$= 525 \text{ cm}^3$ 1½

Remaining volume of box = Volume of cuboidal box

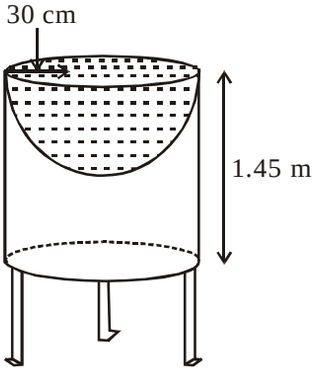
– Volume of 4 conical depressions ½

$= 525 \text{ cm}^3 - 1.464 \text{ cm}^3$

$= 523.5 \text{ cm}^3$

Or

Ramesh made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.



Sol. Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere.

Then, the total surface area = CSA of cylinder + CSA of hemisphere ½

$= 2\pi rh + 2\pi r^2$

$= 2\pi r (h + r)$ 2

$= 2 \times \frac{22}{7} \times 30 (145 + 30) \text{ cm}^2$ 1

$= 2 \times \frac{22}{7} \times 30 \times 175 \text{ cm}^2$ ½

$= 33000 \text{ cm}^2 = 3.3 \text{ m}^2$ 1

35. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	**2**
20 – 25	**4**
25 – 30	**18**
30 – 35	**21**
35 – 40	**33**
40 – 45	**11**
45 – 50	**3**
50 – 55	**6**
55 – 60	**2**

Sol.

Class interval	Number of policy holders (f)	Cumulative frequency (cf)
Below 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45
35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
50 – 55	6	98
55 – 60	2	100

1

$n = 100 \Rightarrow \frac{n}{2} = 50.$

Therefore, median class = 35 – 40,

Class size, $h = 5$, Lower limit of median class, $l = 35$,

Frequency $f = 33$, Cumulative frequency $cf = 45$ ½

$$\Rightarrow \text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$ 1½

$$\Rightarrow \text{Median} = 35 + \left[\frac{50 - 45}{33}\right] \times 5$$ 1

$$\Rightarrow 35 + \frac{25}{33} = 35 + 0.76$$

$$= 35.76$$ 1

Therefore, median age is 35.76 years.

SECTION – E

Case study based questions are compulsory.

36. Case Study – 1

In the months of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021 – 22, as per a report. A car manufacturing company planned to produce 1800 cars in 4th year and 2600 cars in 8th year, assuming that the production increases uniformly by a fixed number every year.

Based on the above information, answer the following questions :

(*i*) Find the production in the 1st year.

(*ii*) Find the production in the 12th year.

(*iii*) Find the total production in first 10 years.

Sol. (*i*) Since the production increases uniformly by a fixed number every year, the number of cars manufactured in 1st, 2nd, 3rd, ..., years will form an AP.

So, $a + 3d = 1800$ and $a + 7d = 2600$ ½

So, $d = 200$ and $a = 1200$. That is production in first year is 1200. ½

(*ii*) $t_{12} = a + 11d \Rightarrow t_{12} = 1200 + 11 \times 200$ ½

$\Rightarrow t_{12} = 3400$ ½

(*iii*) $S_n = \frac{n}{2}[2a + (n-1)\,d] \Rightarrow S_{10} = \frac{10}{2}[2 \times 1200 + (10-1)\,200]$ ½

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 1200 + 9 \times 200]$ ½

$\Rightarrow S_{10} = 5 \times [2400 + 1800]$ ½

$\Rightarrow S_{10} = 5 \times 4200 = 21000$ ½

Or

In which year, the total production will reach to 31200 cars?

Sol. Let in n years, the production will reach to 31200

$S_n = \frac{n}{2}[2a + (n-1)\,d] = 31200 \Rightarrow \frac{n}{2}[2 \times 1200 + (n-1)\,200] = 31200$ ½

$\Rightarrow \frac{n}{2}[2 \times 1200 + (n-1)\,200\,] = 31200 \Rightarrow n[12 + (n-1)] = 312$ ½

$\Rightarrow n^2 + 11n - 312 = 0$

$\Rightarrow n^2 + 24n - 13n - 312 = 0$ ½

$\Rightarrow (n + 24)(n - 13) = 0$

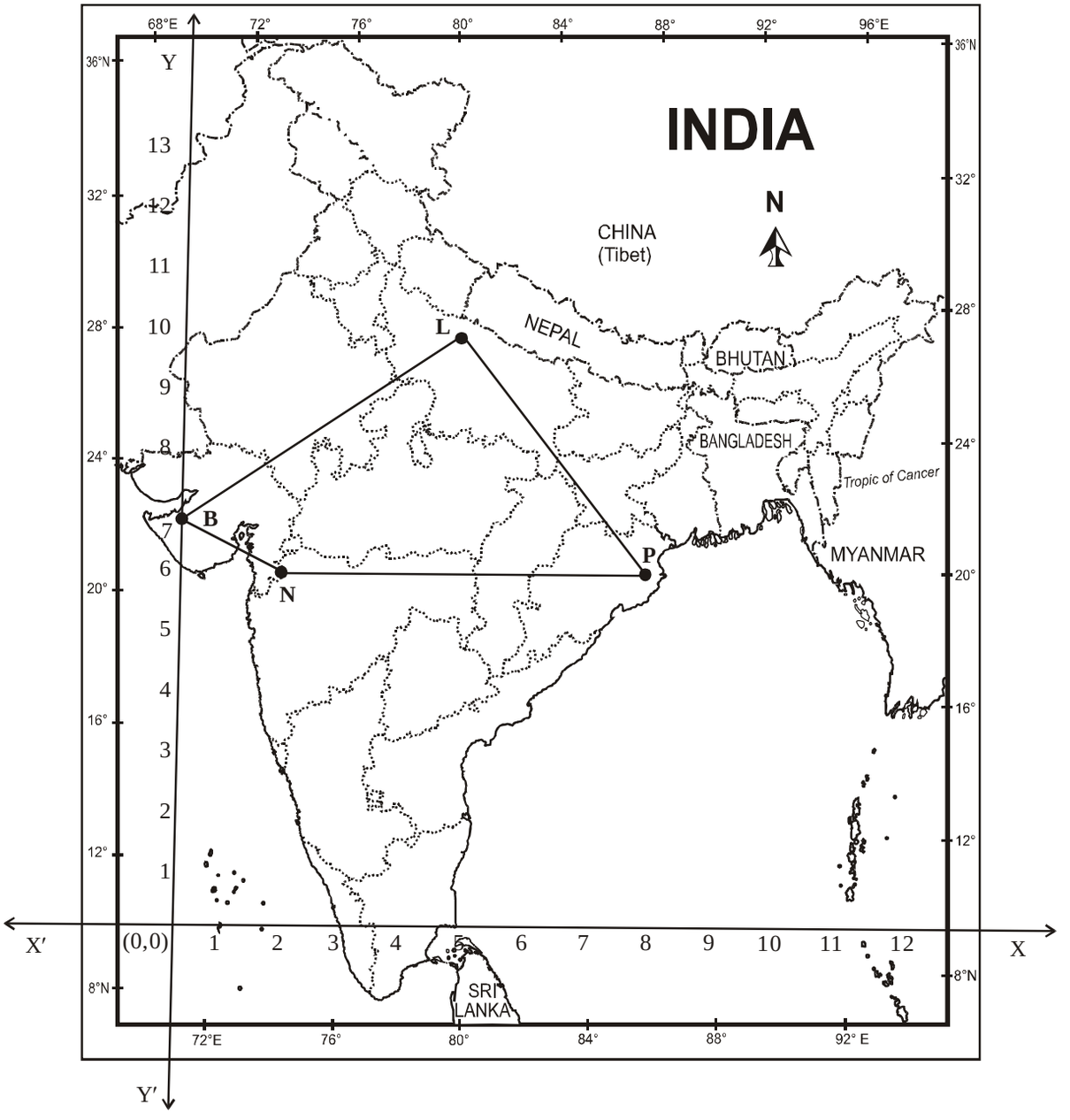
$\Rightarrow n = 13$ or -24.

As n can't be negative. So $n = 13$. ½

37. Case Study – 2

In a GPS, the lines that run east-west are known as lines of latitude, and the lines running north-south are known as lines of longitude. The latitude and the longitude of a place are its coordinates and the distance formula is used to find the distance between two places. The distance between two parallel lines is approximately 150 km. A family from Uttar Pradesh planned a round trip from

Lucknow (L) to Puri (P) via Bhuj (B) and Nashik (N) as shown in the figure given below.



Based on the above information answer the following questions, using the coordinate geometry.

(*i*) Find the distance between Lucknow (L) to Bhuj (B).

(*ii*) If Kota (K), internally divides the line segment joining Lucknow (L) to Bhuj (B) into the ratio 3 : 2, then find the coordinates of Kota (K).

(*iii*) Name the type of triangle formed by the places Lucknow (L), Nashik (N) and Puri (P).

Sol. (*i*) LB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \Rightarrow$ LB = $\sqrt{(0 - 5)^2 + (7 - 10)^2}$ ½

LB = $\sqrt{(5)^2 + (3)^2} \Rightarrow$ LB = $\sqrt{25 + 9} \Rightarrow$ LB = $\sqrt{34}$

Hence, the distance between Lucknow and Bhuj is $150\sqrt{34}$ km. ½

(*ii*) Coordinates of Kota (K) is $\left(\frac{3\times5+2\times0}{3+2}, \frac{3\times7+2\times10}{3+2}\right)$ ½

$= \left(\frac{15+0}{5}, \frac{21+20}{5}\right) = \left(3, \frac{41}{5}\right)$ ½

(*iii*) L(5, 10), N(2, 6), P(8, 6) ½

$LN = \sqrt{(2-5)^2 + (6-10)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$ ½

$NP = \sqrt{(8-2)^2 + (6-6)^2} = \sqrt{(4)^2 + (0)^2} = 4$ ½

$PL = \sqrt{(8-5)^2 + (6-10)^2} = \sqrt{(3)^2 + (4)^2} \Rightarrow LB = \sqrt{9+16} = \sqrt{25} = 5$

as LN = PL ≠ NP, so ΔLNP is an isosceles triangle. ½

Or

Find a place (point) on the longitude (*y*-axis), which is equidistant from the points Lucknow (L) and Puri (P).

Sol. Let A (0, *b*) be a point on the *y*-axis. Then, AL = AP

$\Rightarrow \quad \sqrt{(5-0)^2 + (10-b)^2} = \sqrt{(8-0)^2 + (6-b)^2}$ ½

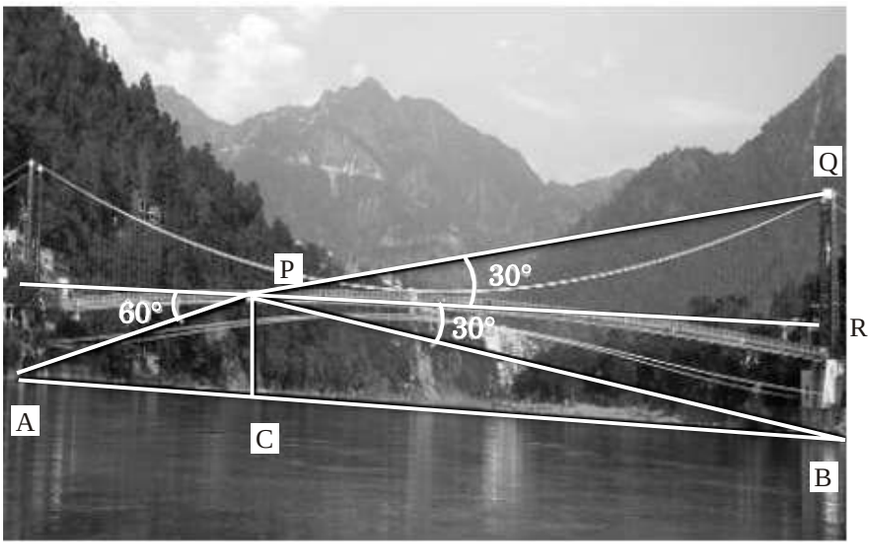
$\Rightarrow \quad (5)^2 + (10-b)^2 = (8)^2 + (6-b)^2$ ½

$\Rightarrow \quad 25 + 100 - 20b + b^2 = 64 + 36 - 12b + b^2 \Rightarrow 8b = 25 \Rightarrow b = \frac{25}{8}$ ½

So, the coordinates on *y*-axis is $\left(0, \frac{25}{8}\right)$ ½

38. Case Study – 3

Lakshaman Jhula is located 5 kilometres north-east of the city of Rishikesh in the Indian state of Uttarakhand. The bridge connects the villages of Tapovan to Jonk. Tapovan is in Tehri Garhwal district, on the west bank of the river, while Jonk is in Pauri Garhwal district, on the east bank. Lakshman Jhula is a pedestrian bridge also used by motorbikes. It is a landmark of Rishikesh. A group of Class X students visited Rishikesh in Uttarakhand on a trip. They observed from a point (P) on a river bridge that the angles of depression of opposite banks of the river are 60° and 30° respectively. The height of the bridge is about 18 metres from the river.



Based on the above information answer the following questions :

(*i*) Find the distance PA.

(*ii*) Find the distance PB.

(*iii*) Find the width AB of the river.

Sol. (*i*) $\sin 60° = \frac{PC}{PA}$ ½

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{18}{PA} \Rightarrow PA = 12\sqrt{3}$ m ½

(*ii*) $\sin 30° = \frac{PC}{PB}$ ½

$\Rightarrow \frac{1}{2} = \frac{18}{PB} \Rightarrow PB = 36$ m ½

(*iii*) $\tan 60° = \frac{PC}{AC} \Rightarrow \sqrt{3} = \frac{18}{AC} \Rightarrow AC = 6\sqrt{3}$ m 1

$\tan 30° = \frac{PC}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{18}{CB} \Rightarrow CB = 18\sqrt{3}$ m ½

Width AB = AC + CB = $6\sqrt{3} + 18\sqrt{3} = 24\sqrt{3}$ m ½

Or

Find the height BQ, if the angle of the elevation from P to Q be 30°.

Sol. RB = PC = 18 m and PR = CB = $18\sqrt{3}$ m ½

$\tan 30° = \frac{QR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{QR}{18\sqrt{3}} \Rightarrow QR = 18$ m 1

QB = QR + RB = 18 + 18 = 36 m. Hence, height BQ is 36 m. ½

8 SAMPLE QUESTION PAPER

MATHEMATICS (Basic)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions :

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Questions of 2 marks, 2 Questions of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. Rahul has 40 cm long red and 84 cm long blue ribbon. He cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece?

(A) 4 cm as it is the LCM of 40 and 84 (B) 4 cm as it is the HCF of 40 and 84

(C) 8 cm as it is the LCM of 40 and 84 (D) 8 cm as it is the HCF of 40 and 84

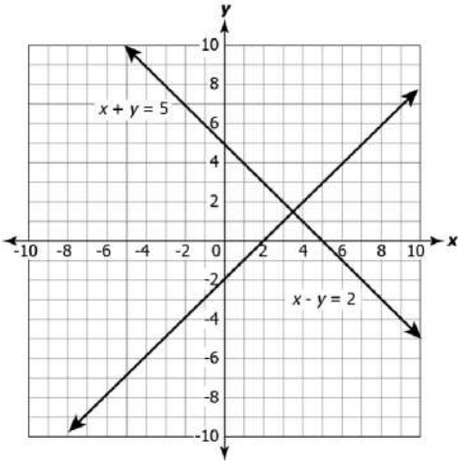
2. Three bulbs red, green and yellow flash at intervals of 80 seconds, 90 seconds and 110 seconds. All three flash together at 8:00 am. At what time will the three bulbs flash altogether again?

(A) 9:00 am (B) 9:12 am (C) 10:00 am (D) 10:12 am

3. What are the roots of the equation $4x^2 - 2x - 20 = x^2 + 9x$?

(A) $\frac{-4}{3}$ and -5 (B) $\frac{4}{3}$ and -5 (C) $\frac{-4}{3}$ and 5 (D) $\frac{4}{3}$ and 5

4. Consider the graph shown.



Which of these is true about the given graph?

(A) These lines have a unique solution as they are intersecting at a point.

(B) These lines have infinitely many solutions as they lie in the same quadrant.

(C) These lines have a unique solution as the coefficient of x in both the equations is one.

(D) These lines have infinitely many solutions as they lie in the same quadrant.

5. What is the smallest positive integer value of k such that the roots of the equation $x^2 - 9x + 18 + k = 0$ can be calculated by factoring the equation?

(A) 1 (B) 2 (C) 3 (D) 4

6. Rahul claims that congruent figures are similar as well. Aman claims that similar figures are congruent as well. Who is/are correct?

(A) only Rahul (B) only Aman

(C) Both Rahul and Aman (D) Neither Rahul nor Aman

7. Consider the statements below.

Statement 1 : All circles are similar.

Statement 2 : All squares are similar.

Statement 3 : All right triangles are congruent.

Statement 4 : All equilateral triangles are congruent.

Which statement is/are correct?

(A) Statement 1 and Statement 3 (B) Statement 2 and Statement 4

(C) Statement 1 and Statement 2 (D) Statement 3 and Statement 4

8. Sheena was asked to plot a point 10 units on the left of the origin and other point 4 units directly above the origin. Which of the following are the two points?

(A) (10,0) and (0, – 4) (B) (–10, 0) and (4, 0)

(C) (10, 0) and (0, 4) (D) (–10, 0) and (0, 4)

9. A circle passes through point P. How many tangents and secants to the circle are possible that pass through P ?

(A) Tangent : 1; Secant : 1 (B) Tangent : Infinite; Secant : 1

(C) Tangent : 1; Secant : Infinite (D) Tangent : Infinite; Secant : Infinite

10. Which of the following completes the statement below ?

The adjacent and _____ of right triangle depends on the $\angle x$ being referred to in that triangle and for the complementary angle $(90 - x)°$ of that triangle, the adjacent and opposite are _____.

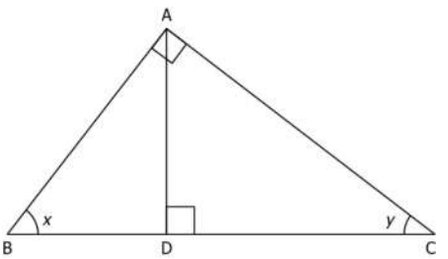
(A) opposite side, reversed (B) opposite side, same

(C) hypotenuse, reversed (D) hypotenuse, same

11. What is the value of $\dfrac{3 - \sin^2 60°}{\tan 30° \tan 60°}$?

(A) $1\frac{1}{4}$ (B) $2\frac{1}{4}$ (C) $2\frac{1}{2}$ (D) $2\frac{3}{4}$

12. Considering the diagram below.



Which of the following statements is true?

(A) Side AC is adjacent to $\angle y$ in triangle ADC and triangle ABC

(B) Side AC is adjacent to $\angle y$ in triangle ADC and side BC is adjacent to $\angle y$ in triangle ABC

(C) Side DC is adjacent to $\angle y$ in triangle ADC and side AC is adjacent to $\angle y$ in triangle ABC
(D) Side AC is adjacent to $\angle y$ in triangle ADC and side DC is adjacent to $\angle y$ in triangle ABC

13. Consider the statements below.
Statement A : A quarter circle represents a sector of the circle.
Statement B : A semicircle represents both sector and segment of the circle.
Which of these statements is correct?
(A) Only Statement A (B) Only Statement B
(C) Both the statements A and B (D) Neither statement A nor B

14. Which of these is equivalent to the sum of the lengths of arc corresponding to the minor and major segment of a circle of radius 12 cm?
(A) 12π cm (B) 24π cm (C) 36π cm (D) 144π cm

15. A medicine-capsule is in the shape of a cylinder of radius 0.25 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. What is the total surface area of the capsule ? (Take p as 3.14)
(A) 3.14 cm^2 (B) 2.7475 cm^2 (C) 0.98125 cm^2 (D) 0.785 cm^2

16. The table below shows the time taken by a group of students to complete 100 m dash.

Time taken (in sec)	Number of students
18-20	3
20-22	18
22-24	26
24-26	19
26-28	9
28-30	5

Which of these is the mean time taken, in sec, by the group of students to complete the 100 m dash when calculated using direct method?
(A) 18.16 (B) 18.96 (C) 22.7 (D) 23.7

17. When calculated using direct method, the mean of the data set shown in the table below is 31.

Class Interval	Frequency
0-10	22
10-20	24
20-30	35
30-40	30
40-50	27
50-60	m
60-70	6
70-80	4

What is the frequency for the class interval 50-60 ?
(A) 10 (B) 12 (C) 20 (D) 24

18. If a card is drawn from a deck of cards, what is the probability of a card drawn to be a red or a black card and what can we say about that event?
(A) 1 and it is a sure event. (B) 0 and it is a sure event.
(C) 1 and it is an impossible event. (D) 0 and it is an impossible event.

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.
(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. (A) : For two positive integers a and b, HCF (a, b) × LCM (a, b) = ab.

(R) : LCM of two numbers is 40, their HCF is 5 and their product is 150.

20. (A) : Distance of the point (4, 7) from y-axis is 4 units.

(R) : Distance of a point from y-axis is given by its x-coordinate.

SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and three more balls of the same kind for ₹ 1300. Represent this situation algebraically.

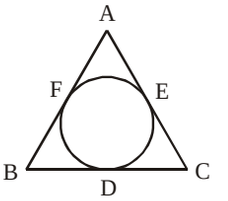
22. A girl of height 100 cm is walking away from the base of a lamp post at a speed of 1.9 m/s. If the lamp is 5 m above the ground, find the length of her shadow after 4 seconds.

Or

If two triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of corresponding angle bisector segments.

23. In the figure given alongside, sides AB, BC and CA of DABC touch a circle at F, D and E respectively. Prove that

$$AF + BD + CE = \frac{1}{2}(\text{Perimeter of DABC}).$$

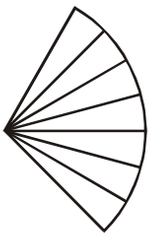


24. Take A = 60° and B = 30°. Write the value of

cos A, cos B and cos(A + B). Is cos (A + B) = cos A + cos B ?

25. A Japanese fan can be made by sliding open its 7 small sections (or leaves) which are each in the form of sector of a circle having central angle of 15°. If the radius of this fan is 24 cm, find the area of this fan, while opened (See figure).

$\left(\text{Use } \pi = \frac{22}{7}\right)$



Or

Find the area of the sector of a circle whose radius is 6 cm and the length of the corresponding arc is 12 cm.

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Find the LCM and HCF of 26, 72 and 108 by prime factorisation method.

27. If one zero of the polynomial $ax^2 + 15x + 6$ is reciprocal of the other, then find the value of a. Also, find the zeroes of the polynomial.

28. Solve by elimination : $5x - 2y = 11$; $3x + 4y = 4$

Or

Solve for x and y : $3x + 4y = 13$; $2x - 3y = 3$

29. Three concentric circles with centre O and radii r_1, r_2, r_3 are such that $r_1 < r_2 < r_3$. P is a point on the circle with radius r_2 and Q is a point on circle with radius r_3. How many tangents can be drawn from P and Q to the circles ?

30. Prove that :

$$\frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} = 2\text{cosec } \alpha.$$

Or

Prove that : $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$.

31. A letter is chosen at random from the letters of the message 'HEY DUDE WASSUP!'. What is the probability that the chosen letter is a consonant ?

SECTION – D

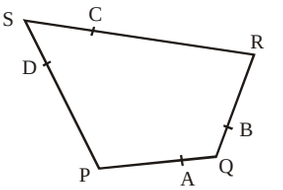
Section-D consists of 4 questions of 5 marks each.

32. A motor boat whose speed is 18 km/h in still water takes 1 h more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Or

A train travels at a certain average speed for a distance of 63 km and then travels for a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed ?

33. In the figure, A, B, C and D are points on the sides of a quadrilateral PQRS such that these points divide the sides PQ, RQ, RS and PS in the ratio 2 : 1. Prove that ABCD is a parallelogram.



34. Water flows through a cylindrical pipe, whose inner diameter is 7 cm, at the rate of 6 km/h in an empty cylindrical tank, the radius of whose base is 40 cm and height is 4.9 m. How long will it take to fill the whole tank ?

Or

Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm ?

35. Find the median and mode of the following data :

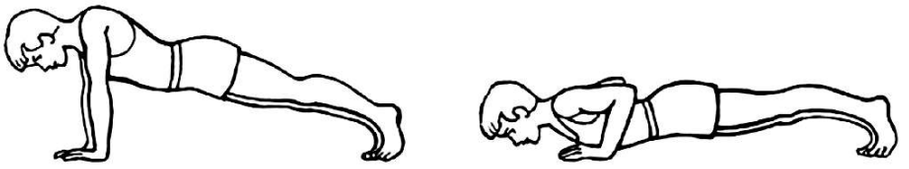
Class interval	Frequency
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

SECTION – E

Case study based questions are compulsory.

36. Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest,

arms, and shoulders, support required from other muscles helps in toning up the whole body.



Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour. But he wants to achieve a target of 3900 push-ups in 1 hour for which he practices regularly. With each day of practice, he is able to make 5 more push-ups in one hour as compared to the previous day. If on first day of practice, he makes 3000 push-ups and continues to practice regularly till his target is achieved. Keeping the above situation in mind, answer the following questions :

(*i*) Form an A.P. representing the number of push-ups per day. (1)

(*ii*) Find the number of push-ups performed by Nitesh on the 4th day. (1)

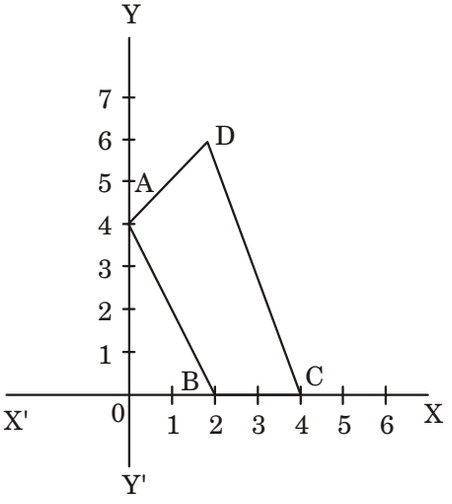
(*iii*) Find the minimum number of days he needs to practice before the day his goal is accomplished. (2)

Or

Find the total number of push-ups performed by him upto the day his goal is achieved. (2)

37. Four friends Ashok, Balbir, Chandra and David are playing with a ball in a park. One student Mohan of class X was just sitting in the park to complete his mathematics project. He plots their positions A, B, C and D respectively as shown in the figure given below. Mohan is sitting at O.

Based on the above information, answer the following questions :



(*i*) What are coordinates of Ashok and Balbir ?

(*ii*) What are coordinates of Chandra and David ?

(*iii*) Find the distance between Ashok and Balbir ?

Or

Find the distance of David from Mohan ?

38. Qutub Minar, located in South Delhi, India was built in the year 1193. It is 72 m high tower. Working on a school project, Charu and Daljeet visited the monument. They used trigonometry to find their distance from the tower. Observe the picture given below. Points

C and D represent their positions on the ground in line with the base of tower, the angles of elevation of top of the tower (point A) are 60° and 45° from points C and D respectively.

(*i*) Based on above information, draw a well-labelled diagram.

(*ii*) Find the distance BD from the base of the tower.

(*iii*) Find the distance BC.

Or

What is the distance between points C and A ?

Answers

1. (B) 4 cm as it is the HCF of 40 and 84 **2.** (D) 10:12 am

3. (C) $\frac{-4}{3}$ and 5 **4.** (A) **5.** (B) 2 **6.** (A) only Rahul

7. (C) Statement 1 and Statement 2 **8.** (D) (–10, 0) and (0, 4)

9. (C) Tangent : 1; Secant : Infinite

10. (A) opposite side, reversed (B) opposite side, same **11.** (B) $2\frac{1}{4}$

12. (C) Side DC is adjacent to $\angle y$ in triangle ADC and side AC is adjacent to $\angle y$ in triangle ABC

13. (C) Both the statements A and B **14.** (B) 24π cm **15.** (D) 0.785 cm^2

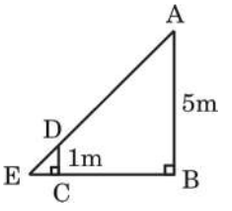
16. (D) 23.7 **17.** (B) 12 **18.** (A) 1 and it is a sure event.

19. (C) (A) is true but (R) is false.

20. (A) Both (A) and (R) are true and (R) is the correct explanation of (A).

21. Let cost of one bat = ₹ x and cost of one ball = ₹ y.
According to the question,
By first condition, $3x + 6y = 3900$ ...(*i*)
By second condition, $x + 3y = 1300$ ...(*ii*)
which is required algebraic form.

22. Speed of girl = 1.9 m/second.
After 4 seconds, distance covered by the girl
= 1.9 × 4 m = 7.6 m.
AB = Lamp post = 5 m
CD = Height of girl
= 100 cm
= 1 m.



EC = Shadow of girl = ?

$\angle B = \angle C = 90°$ [Each]

Now, $\angle E = \angle E$ [Common angle]

So, $\Delta ABE \sim \Delta DCE$ [AA similarity]

Hence, $\frac{AB}{CD} = \frac{BE}{EC}$.

$\Rightarrow \quad \frac{5}{1} = \frac{7.6 + EC}{EC}$.

or $5EC = 7.6 + EC$

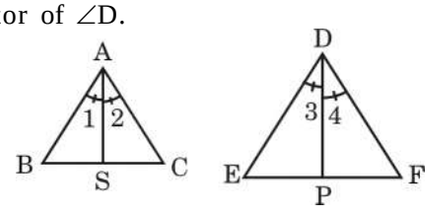
$\Rightarrow \quad 4EC = 7.6$

$\Rightarrow \quad EC = \frac{7.6}{4} = 1.9$ m.

Hence, length of shadow is 1.9 m.

Or

Given : In ΔABC and ΔDEF, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and AS is the bisector of $\angle A$ and DP is the bisector of $\angle D$.



To prove : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AS}{DP}$

Proof : In ΔABC and ΔDEF, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$.

$\Rightarrow \quad \Delta ABC \sim \Delta DEF$ [AAA similarity]

$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ...(*i*) [Corresponding sides]

Now, in ΔABS and ΔDEP,

$\angle B = \angle E$ [Given]

and $\angle A = \angle D$ [Given]

or $\frac{1}{2}\angle A = \frac{1}{2}\angle D$

So, $\Delta ABS \sim \Delta DEP$ [AA similarity]

Therefore, $\frac{AB}{DE} = \frac{AS}{DP}$ ...(*ii*) [Corresponding sides]

From (*i*) and (*ii*), we get

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AS}{DP}$.

Hence, proved.

23. We have :

Perimeter of ΔABC = AB + BC + AC.

= AF + BF + BD + CD + AE + CE.

= AF + BD + BD + CE + AF + CE

[Since AF = AE, BD = BF and CD = CE]

= 2AF + 2BD + 2CE

= 2(AF + BD + CE).

So, $AF + BD + CE = \frac{1}{2}$(Perimeter of ΔABC).

Hence, proved.

24. $\cos A = \cos 60° = \frac{1}{2}$,

$\cos B = \cos 30° = \frac{\sqrt{3}}{2}$

and $\cos (A + B) = \cos (60° + 30°) = \cos 90° = 0.$

So, $\cos (A + B) \neq \cos A + \cos B.$

25. Area of the fan = Area of the 7 small leaves

$$= 7 \times \frac{15°}{360°} \times \pi \times (24)^2 \text{ cm}^2.$$

$$= 7 \times \frac{1}{24} \times \frac{22}{7} \times 24 \times 24 \text{ cm}^2$$

$$= 528 \text{ cm}^2.$$

Or

$$l = \frac{2\pi r\theta}{360°} \Rightarrow 12 = 2\pi \times 6 \times \frac{\theta}{360°}.$$

$$\Rightarrow \frac{\theta}{360°} = \frac{1}{\pi}.$$

So, area of the sector $= \frac{\pi r^2 \theta}{360°} = \pi \times (6)^2 \times \frac{1}{\pi} = 36 \text{ cm}^2.$

Alternate method :

$$\text{Area} = \frac{l}{2\pi r} \times \pi r^2.$$

$$= \frac{12}{2\pi \times 6} \times \pi \times 36 = 36 \text{ cm}^2.$$

26. We have : $26 = 2 \times 13.$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

and $108 = 2 \times 2 \times 3 \times 3 \times 3.$

So, $\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 13$

$= 8 \times 27 \times 13 = 2808.$

and $\text{HCF} = 2.$

27. ***Hint :*** Product of zeroes $= \frac{c}{a}$ will be used.

We have : $ax^2 + 15x + 6.$

Let one zero be α.

Then, the other zero will be $\frac{1}{\alpha}$.

So, $\alpha \times \frac{1}{\alpha} = \frac{6}{a}$

$\Rightarrow a = 6.$

So, the polynomial is

$6x^2 + 15x + 6 = 6x^2 + 12x + 3x + 6$

$= 6x (x + 2) + 3 (x + 2)$

$= (x + 2) (6x + 3)$

Now, $(x + 2) = 0$ gives $x = -2$

and $6x + 3 = 0$ gives $x = \frac{-3}{6} = \frac{-1}{2}.$

28. Given pair of linear equations is

$$5x - 2y = 11 \quad ...(i)$$

and $$3x + 4y = 4 \quad ...(ii)$$

Using method of elimination by equating coefficients, we proceed as follows :

Multiplying eqn. (*i*) by 2, we get

$$10x - 4y = 22 \quad ...(iii)$$

Adding (*ii*) and (*iii*), we get

$$13x = 26 \Rightarrow x = 2.$$

Putting $x = 2$ in eqn. (*i*), we get

$$5(2) - 2y = 11 \Rightarrow 10 - 2y = 11$$

or $$-2y = 11 - 10$$

or $$-2y = 1 \Rightarrow y = \frac{-1}{2}.$$

So, $x = 2$ and $y = \frac{-1}{2}$ is the required solution.

Or

We have :

$$3x + 4y = 13 \quad ...(1)$$

$$2x - 3y = 3 \quad ...(2)$$

Multiplying (1) by 3 and (2) by 4, we get

$$9x + 12y = 39 \quad ...(3)$$

$$8x - 12y = 12 \quad ...(4)$$

Adding (3) and (4), we get

$$17x = 51.$$

$$\Rightarrow x = \frac{51}{17} = 3.$$

Putting $x = 3$ in (1), we get

$$3 \times 3 + 4y = 13.$$

$$\Rightarrow 4y = 13 - 9 = 4$$

$$\Rightarrow y = \frac{4}{4} = 1.$$

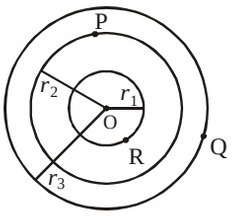
Thus, $x = 3$ and $y = 1$.

29. From P, two tangents can be drawn to the circle with radius r_1, because it is outside the circle.

From P, one tangent can be drawn to the circle with radius r_2 as it is on the circle.

From P, no tangent can be drawn to the circle with radius r_3 as it lies inside the circle.

From Q, two tangents can be drawn to each of circles with radius r_1 and r_2, because it lies outside of both the circles.



From Q, only one tangent can be drawn to the circle with radius r_3 as it lies on it.

30.

$$\text{LHS} = \frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha}$$

$$= \frac{\sin^2\alpha + (1+\cos\alpha)^2}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{\sin^2\alpha + 1 + \cos^2\alpha + 2\cos\alpha}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{1 + 1 + 2\cos\alpha}{\sin\alpha(1+\cos\alpha)} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2 + 2\cos\alpha}{\sin\alpha(1+\cos\alpha)} = \frac{2(1+\cos\alpha)}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{2}{\sin\alpha} = 2 \times \frac{1}{\sin\alpha}$$

$$= 2 \text{ cosec } \alpha = \text{RHS}.$$

Hence, proved.

Or

$$\text{LHS} = \frac{1-\sin\theta}{1+\sin\theta} = \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= (\sec\theta - \tan\theta)^2 = \text{RHS}.$$

Hence, proved.

31. ***Hint :*** LHS will be multiplied and divided by $(1 - \sin\theta)$ and then simplified.

Total possible outcomes = 13.

Outcomes favourable to a consonant are : H, Y, D, D, W, S, S, P (Total 8).

So, required probability = $\frac{8}{13}$.

32. ***Hint :*** Quadratic equation will be formed, using given condition. It will be solved using quadratic solution will be reflected.

Let the speed of the stream be x km/h.

Then, the speed of the motor boat downstream = $(18 + x)$ km/h

and the speed of the motor boat upstream = $(18 - x)$ km/h.

Then, time taken by boat going downstream = $\frac{24}{18+x}$ hours.

and time taken by it going upstream = $\frac{24}{18-x}$ hours.

Then, according to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1.$$

$$\Rightarrow \quad 24\left[\frac{18 + x - (18 - x)}{(18-x)(18+x)}\right] = 1.$$

$$\Rightarrow \quad \frac{24 \times 2x}{(18)^2 - x^2} = 1.$$

$$\Rightarrow \quad 48x = 324 - x^2.$$

$$\Rightarrow \quad x^2 + 48x - 324 = 0.$$

$\Rightarrow \quad x = \dfrac{-48 \pm \sqrt{48 \times 48 - 4 \times 1(-324)}}{2}$

$x = \dfrac{-48 \pm 12\sqrt{25}}{2}$

$\Rightarrow \quad x = \dfrac{-48 \pm 60}{2}$

$\Rightarrow \quad x = -54$ or $x = 6$.

Neglecting the negative value of x, we have $x = 6$.

So, the speed of the stream is 6 km/h.

Or

Hint : Quadratic equation will be formed, using time and distance.

Let the original speed of the train be x km/h.

Then, the time taken by the train to cover 63 km with the speed of x km/h = $\dfrac{63}{x}$ hours.

The higher speed of the train is $(x + 6)$ km/h.

Then, the time taken to travel 72 km/h = $\dfrac{72}{x+6}$ hours.

According to the question,

$$\frac{63}{x} + \frac{72}{x+6} = 3.$$

$$\frac{63(x+6) + 72x}{x(x+6)} = 3.$$

$$63x + 63 \times 6 + 72x = 3x^2 + 18x.$$

$\Rightarrow \quad 135x + 378 = 3x^2 + 18x$

$\Rightarrow \quad 3x^2 + 18x - 135x - 378 = 0.$

$\Rightarrow \quad 3x^2 - 117x - 378 = 0.$

$\Rightarrow \quad x^2 - 39x - 126 = 0.$

$\Rightarrow \quad x = \dfrac{39 \pm \sqrt{39 \times 39 - 4 \times 1 \times (-126)}}{2}$

$\Rightarrow \quad x = \dfrac{39 \pm 3\sqrt{169 + 56}}{2}$

$\Rightarrow \quad x = \dfrac{39 \pm 3 \times 15}{2}$

$\Rightarrow \quad x = \dfrac{39 \pm 45}{2}$

$\Rightarrow \quad x = 42$ or $x = -3$.

Neglecting the negative value of x, we get $x = 42$.

Hence, original average speed of the train is 42 km/h.

33. ***Hint :*** After joining QS, converse of BPT shall be applied in four triangles.

Join PR.

In ΔQPR, $\dfrac{QA}{PA} = \dfrac{QB}{BR} = 1:2$.

So, by converse of BPT,

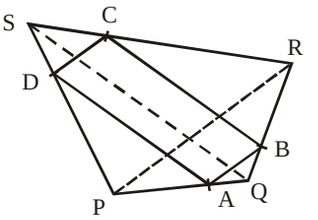
$AB \parallel PR$ ...(1)

Similarly, in ΔSRP, we get

$CD \parallel PR$ ...(2)

From (1) and (2), we get

$$AB \parallel CD \qquad ...(3)$$



Similarly, by joining QS, we can get

$$BC \parallel AD \qquad ...(4)$$

From (3) and (4), opposite sides of quadrilateral ABCD are parallel.
So, ABCD is a parallelogram.

34. Volume of the cylindrical tank $= \pi \times \left(\frac{40}{100}\right)^2 \times 4.9\ m^3$.

Further, speed of water = 6 km/h
= 6000 m per hour
$= \frac{6000}{60}$
= 100 m per minute.

Let the tank will be filled in x minutes.
So, we have :

$$x \times \pi \times \left(\frac{7}{2 \times 100}\right)^2 \times 100 = \pi \times \left(\frac{40}{100}\right)^2 \times 4.9.$$

$$\Rightarrow \quad x \times \pi \times \frac{49}{4 \times 10000} \times 100 = \pi \times \frac{40 \times 40 \times 4.9}{100 \times 100}$$

$$\Rightarrow \quad \frac{x \times \pi \times 49}{400} = \frac{\pi \times 16}{100} \times \frac{49}{10}.$$

$$\Rightarrow \quad x = \frac{\pi \times 16 \times 49 \times 400}{100 \times 10 \times \pi \times 49}$$

$$= \frac{16 \times 4}{10}.$$

$$\Rightarrow \quad x = 6.4 \text{ minutes.}$$

Or

Hint : Assuming that water will rise 21 cm in x minutes, volume of water filled in x minutes is calculated. Equating this volume with volumes of the pond, x is calculated.
Let the water will rise 21 cm in x minutes.

Now, volume of water in the pond $= 50 \times 44 \times \frac{21}{100}\ m^3$ ...(1)

Volume of water filled through pipe in one minute $= \pi r^2 h = \frac{22}{7} \times \left(\frac{14}{2 \times 100}\right)^2 \times \frac{15000}{60}\ m^3$.

So, water filled in x minutes $= x \times \frac{22}{7} \times \left(\frac{14}{200}\right)^2 \times \frac{15000}{60}\ m^3$ ...(2)

From (1) and (2),

$$x \times \frac{22}{7} \times \left(\frac{14}{200}\right)^2 \times \frac{15000}{60} = 50 \times 44 \times \frac{21}{100}.$$

$$\Rightarrow \quad x \times \frac{22}{7} \times \frac{49}{10000} \times \frac{15000}{60} = 50 \times 44 \times \frac{21}{100}.$$

$$\Rightarrow \quad x = \frac{50 \times 44 \times 21 \times 7 \times 60 \times 10000}{100 \times 22 \times 49 \times 15000}.$$

$$= \frac{3 \times 60 \times 10000}{15000} = \frac{600000}{5000}$$

$$= \frac{600}{5} \text{ minutes}$$

$$= 120 \text{ minutes}$$

$$= 2 \text{ hours.}$$

35. We have :

Class interval	Frequency	Cumulative frequency
0 – 20	6	6
20 – 40	8	14
40 – 60	10	24
60 – 80	12	36
80 – 100	6	42
100 – 120	5	47
120 – 140	3	50
Total	50	

The median class is 60 – 80, since $\frac{N}{2} = \frac{50}{2} = 25$.

Now, $$\text{median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

Here, $l = 60$, $N = 50$, $cf = 24$, $f = 12$ and $h = 20$.

So, $$\text{median} = 60 + \left(\frac{\frac{50}{2} - 24}{12}\right) \times 20.$$

$$= 60 + \frac{1 \times 20}{12}$$

$$= 60 + \frac{5}{3} = 60 + 1.67.$$

$$= 61.67 \text{ (approx.)}$$

The modal class is also 60 – 80.

Now, $$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h.$$

Here, $l = 60$, $f_1 = 12$, $f_0 = 10$, $f_2 = 6$ and $h = 20$.

$\Rightarrow$ $\text{Mode} = 60 + \left(\frac{12-10}{2\times 12-10-6}\right)\times 20.$

$= 60 + \frac{2\times 20}{8} = 60 + 5 = 65.$

36. (*i*) Required AP is 3000, 3005, 3010, ...

(*ii*) Here, $a = 3000$ and $d = 5$

So, on fourth day, number of push-ups = $a + 3d = 3000 + 3 \times 5 = 3015$.

(*iii*) Let number of days be n.

So, $3900 = 3000 + (n - 1) \times 5 \Rightarrow 900 = 5n - 5$.

$\Rightarrow 5n = 905 \Rightarrow n = 181$.

So, required days = 181.

Or

From $S = \frac{n}{2}\{2a + (n-1)d\}$, we get

$= \frac{181}{2}(2 \times 3000 + 18 \times 5)$

$= 181\,(3000 + 9 \times 5) = 181 \times 3450 = 624450.$

37. (*i*) Ashok (0, 4); Balbir (2, 0).

(*ii*) Chandra (4, 0); David (2, 6).

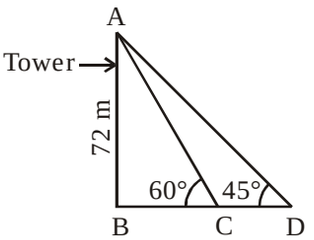
(*iii*) Distance between Ashok and Balbir = $\sqrt{(0-2)^2 + (4-0)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ units.

Or

Distance of David from Mohan = $\sqrt{(2-0)^2 + (6-0)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$ units.

38. (*i*) Diagram is as shown below :



(*ii*) From ΔABD, $\frac{AB}{BD} = \tan 45° \Rightarrow \frac{72}{BD} = 1 \Rightarrow BD = 72$ m.

(*iii*) From ΔDBC, $\frac{AB}{BC} = \tan 60° \Rightarrow \frac{72}{BC} = \sqrt{3} \Rightarrow BC = \frac{72}{\sqrt{3}} = 24\sqrt{3}$ m.

Or

From ΔABC, $\frac{AB}{AC} = \sin 60° \Rightarrow \frac{72}{AC} = \frac{\sqrt{3}}{2} \Rightarrow AC = \frac{144}{\sqrt{3}} = 48\sqrt{3}$ m.

9 SAMPLE QUESTION PAPER

MATHEMATICS (Basic)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions : Same as in CBSE Sample Question Paper.

SECTION – A

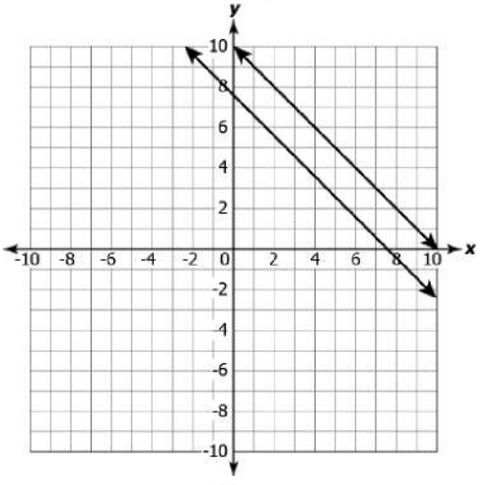
Section-A consists of 20 questions of 1 mark each.

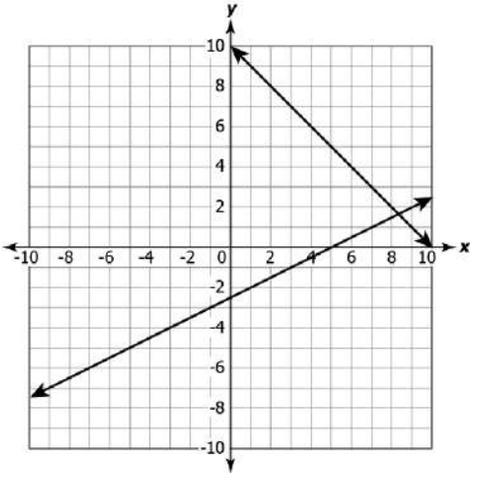
1. Given that HCF (253, 440) = 11 and LCM (253, 440) = 253 × R. The value of R is :
(A) 400 (B) 40 (C) 440 (D) 253

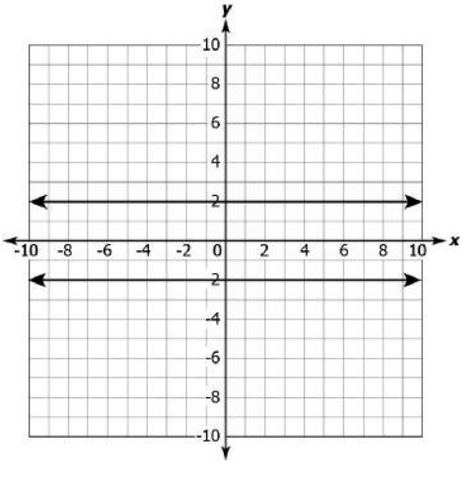
2. If d = LCM (36, 198), then the value of d is :
(A) 396 (B) 198 (C) 36 (D) 1

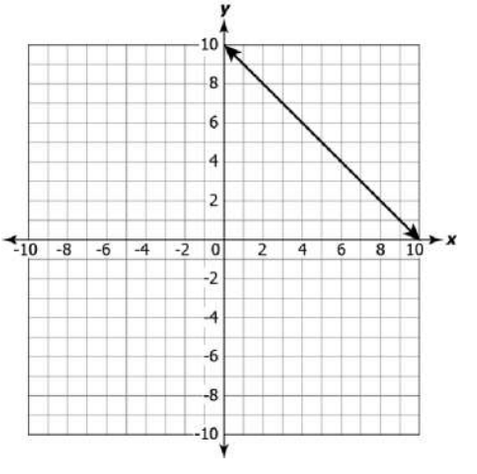
3. A student is trying to find the roots of $3x^2 - 10x - 8 = 0$ by splitting the middle term as follows : Step 1 : $3x^2 - 10x - 8 = 0$ Step 2 : $3x^2 - mx + nx - 8 = 0$
What could be the values of m and n?
(A) $m = 12$ and $n = 2$ (B) $m = -12$ and $n = -2$
(C) $m = 8$ and $n = 2$ (D) $m = -8$ and $n = -2$

4. Which of these linear equations have a unique solution?

(A) 

(B) 

(C) 

(D) 

5. Rahul follows the steps shown below to find the roots of the equation $3x^2 - 11x - 20 = 0$, by splitting the middle term.

Step 1 : $3x^2 - 11x - 20 = 0$

Step 2 : $3x^2 - 15x + 4x - 20 = 0$

Step 3 : $3x\,(x - 5) + 4\,(x - 5) = 0$

Step 4 : $(3x - 4)(x - 5) = 0$

Step 5 : $x = \frac{4}{3}$ and 5

In which step did Rahul make the first error?

(A) Step 2 (B) Step 3 (C) Step 4 (D) Step 5

6. Two quadrilaterals are such that their diagonals bisect each other. What additional information is required to conclude that the quadrilaterals are similar?

(A) Opposite sides are equal

(B) Opposite angles are equal

(C) Diagonals are equal and opposite sides are equal

(D) Diagonals bisect at right angle and adjacent angles are equal

7. Nishant is 6 feet tall. At an instant, his shadow is 5 feet long. At the same instant, the shadow of a tower is 30 feet long. How tall is the tower?

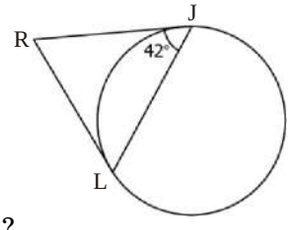
(A) 24 feet (B) 30 feet (C) 36 feet (D) 18 feet

8. Three points lie on a vertical line. Which of the following could be those points?

(A) (0, 4), (4, 0), (0, 0) (B) (4, 3), (5, 3), (– 12, 3)

(C) (– 8, 7), (– 8, – 8), (– 8, – 100) (D) (– 8, 3), (– 8, 8), (8, 7)

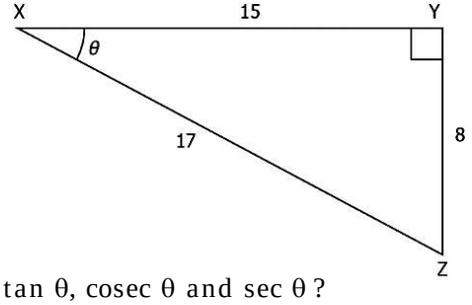
9. In the figure shown below, RJ and RL are tangents of the circle.



What is the measure of $\angle JRL$?

(A) 90° – 42° (B) 90° – 84° (C) 180° – 42° (D) 180° – 84°

10. Consider the triangle shown below.



What are the values of tan θ, cosec θ and sec θ ?

(A) $\tan\theta = \frac{8}{15}, \text{cosec}\,\theta = \frac{17}{15}, \sec\theta = \frac{17}{8}$ (B) $\tan\theta = \frac{8}{15}, \text{cosec}\,\theta = \frac{17}{8}, \sec\theta = \frac{17}{15}$

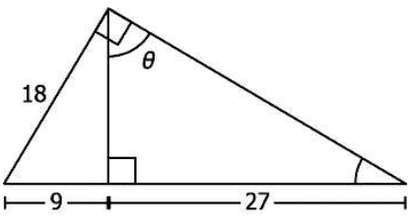
(C) $\tan\theta = \frac{17}{15}, \text{cosec}\,\theta = \frac{8}{15}, \sec\theta = \frac{17}{8}$ (D) $\tan\theta = \frac{8}{15}, \text{cosec}\,\theta = \frac{17}{15}, \sec\theta = \frac{8}{17}$

11. The value of $\frac{4 - \sin^2 45°}{\cot k \tan 60°}$ is 3.5.

What is the value of k ?

(A) 30° (B) 60° (C) 45° (D) 90°

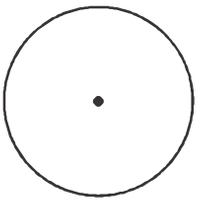
12. Observe the figure shown.



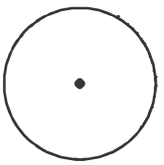
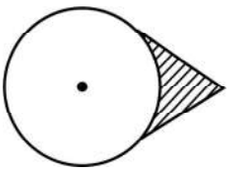
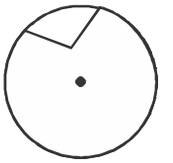
Which of these is the value of cos θ ?

(A) $\frac{1}{2}$ (B) $\frac{2}{1}$ (C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{3\sqrt{3}}{2}$

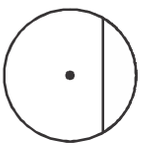
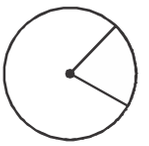
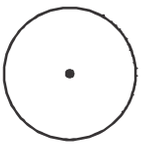
13. Consider a circle given below :



Aman shades a part in the circle which is enclosed by two radii and its corresponding arc. Which of these could he have drawn?

(A)  (B)  (C)  (D) 

14. Which of the following options represents the shaded region as the major sector and unshaded region as the minor sector?

(A)  (B)  (C)  (D) 

15. Arman wants to polish the object, which is composed of a cylinder surmounted by a hemisphere. If the whole length of the solid is 5 m and the diameter of the hemisphere is 7 m, what is the cost of polishing the surface area of the solid at the rate of 50 paise per sq. m. ? $\left(\text{Use } \pi = \frac{22}{7}\right)$

(A) ₹ 1100 (B) ₹ 550 (C) ₹ 110 (D) ₹ 55

16. The table below summarizes the data about the heights of students in a class.

Height (in cm)	Number of students
130-140	6
140-150	14
150-160	20
160-170	12
170-180	8

When calculated using assumed mean method what is the mean height of students in the class?

(A) 154.33 cm (B) 154.67 cm (C) 155.33 cm (D) 155.67 cm

17. A data set is shown.

Class interval	Frequency
4 – 6	2
6 – 8	6
8 – 10	20
10 – 12	28
12 – 14	12
14 – 16	10
16 – 18	2

What is the mean and mode of the data shown?

(A) Mean : 10; Mode : 10.33 (B) Mean : 10; Mode : 10.67

(C) Mean : 11; Mode : 10.33 (D) Mean : 11; Mode : 10.67

18. Tanvi has a box containing four cards labelled A, B, C and D. She randomly picks a card from the box, records the label on the card and put it back in the box. She repeats this experiment 80 times and records her observation in the table shown below.

Card A	Card B	Card C	Card D
11	16	25	28

Which of the following shows the empirical probability and theoretical probability of picking Card C the next time?

(A) Empirical probability = $\frac{5}{16}$; Theoretical probability = $\frac{1}{4}$

(B) Empirical probability = $\frac{5}{16}$; Theoretical probability = $\frac{1}{2}$

(C) Empirical probability = $\frac{5}{11}$; Theoretical probability = $\frac{1}{4}$

(D) Empirical probability = $\frac{5}{11}$; Theoretical probability = $\frac{1}{2}$

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. **(A) :** For any integer n, 15^n can never ends with the digit 0.

(R) : Only prime factors of 15 are 3 and 5.

20. **(A) :** Points A(–4, 0), B(4, 0) and C(0, 3) are vertices of an isosceles triangle.

(R) : Numerical values of x-coordinates of A and B are the same.

SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. Given the linear equation $3x - 4y - 7 = 0$, write another linear equation in these two variables such that the geometrical representation of the pair so formed is :

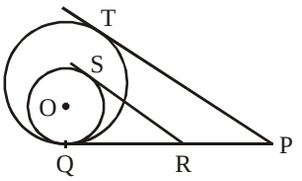
(*i*) intersecting lines, (*ii*) parallel lines.

22. ABCD is a trapezium, in which AB is parallel to DC and its diagonals intersect each other at point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Or

M and N are points on the sides PQ and PR respectively of a ΔPQR. If PN = 4.8 cm, NR = 1.6 cm, PM = 4.5 cm and MQ = 1.5 cm, then find whether MN $\parallel$ QR or not.

23. In the figure, PQ is a common tangent to the two circles touching each other internally at Q. SR and PT are tangents to smaller and larger circles respectively. If SR = 4 cm and PT = 7 cm, find RP.

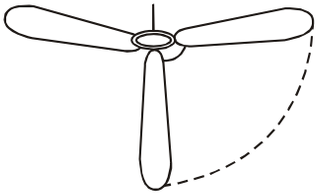


24. Find the value of sin 60° . cos 30° – cos 60° . sin 30°.

25. If the area of a sector of a circle is $\frac{5}{18}$ th of the area of that circle, then find the central angle of the sector.

Or

A ceiling fan has three wings as shown in the figure. Find the length of the arc described between two consecutive wings, where length of each wing is 0.98 m.



SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Prove that $2\sqrt{3} - 7$ is an irrational.

27. If a and b are zeroes of the polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

28. Two chairs and three tables cost ₹ 5650, whereas three chairs and two tables cost ₹ 7100. Find the cost of a chair and a table separately.

Or

Solve by elimination : $9x + 10y = 29$
$10x + 9y = 28$

29. Prove that the lengths of tangents drawn from an external point to a circle are equal.

30. Prove the identity : $\frac{\sin x - \tan x}{\sin x \,.\, \tan x} = \frac{\cos x - 1}{\sin x}$

Or

Prove the identity : $\sec^2 \theta (\sec^2 \theta - 2) + 1 = \tan^4 \theta$.

31. One card is drawn at random from a well shuffled pack of 52 cards. Find the probability of drawing : (*a*) Neither an ace nor a king (*b*) A non spade (*c*) A queen of black suit.

SECTION – D

Section-D consists of 4 questions of 5 marks each.

32. An aeroplane left 40 minutes late due to heavy rains and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km/hour from its original speed. Find the original speed of the aeroplane.

Or

300 oranges are distributed equally among a certain number of students. Had there been 10 more students, each would have received 1 orange less. Find the number of students.

33. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

34. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 2.1 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical bucket full of water in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 9.8 cm, find the volume of water left in the cylindrical bucket. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Or

A decorative pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold pins and paper strips respectively. The dimensions of the cuboid are 20 cm × 15 cm × 10 cm. The radius of each conical depression is 0.5 cm and depth 2.1 cm. The edge of cubical depression is 9 cm. Find the volume of wood used in making the entire pen stand. $\left(\text{Use } \pi = \frac{22}{7}\right)$

35. The literacy rate of females in 50 cities is given in the following frequency distribution :

Literacy rate (in %)	No. of cities
20 – 30	3
30 – 40	2
40 – 50	6
50 – 60	15
60 – 70	8
70 – 80	7
80 – 90	5
90 – 100	4

Find the mode and median of this data.

SECTION – E

Case study based questions are compulsory.

36. We can find A.P. in many situations in our day-to-day life. One such example is a tissue paper roll, in which the first term is the diameter of the core of the roll and twice the thickness of the paper is the common difference. If the sum of first n rolls of tissue on a roll is $S_n = 0.1n^2 + 7.9n$, then answer the following questions :

(*i*) What is the diameter of the core of the roll ?

(*ii*) What is the thickness of the paper ?

(*iii*) Find S_{n-1}.

Or

Find $(S_n - S_{n-1})$.

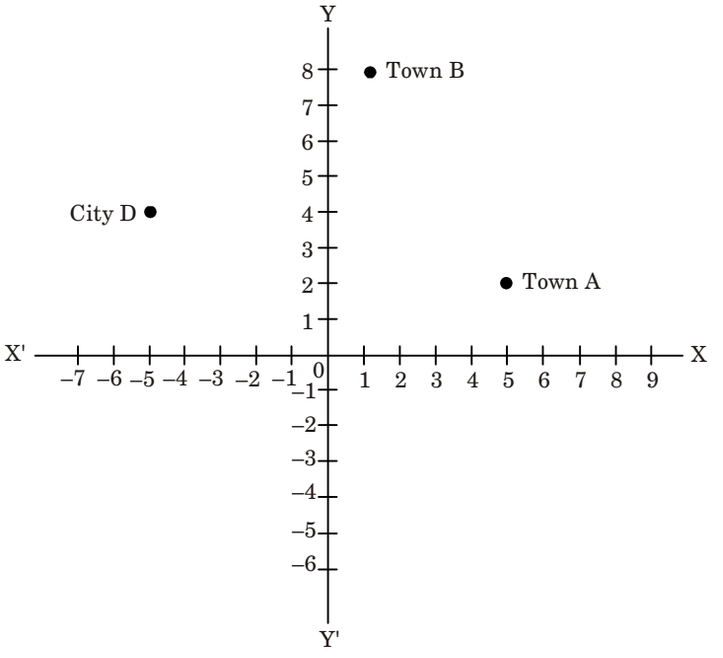
37. Two friends Meena and Anjali were working in the same office at a certain city D. During Deepawali vacations, they both decides to go their hometowns represented by Town A and Town B respectively as shown in the figure.

Based on the above information, answer the following questions :

(*i*) What are coordinates of the point representing hometown of Meena ?

(*ii*) What are coordinates of the point representing hometown of Anjali ?

(*iii*) Find the distance travelled by Meena to reach her hometown ?



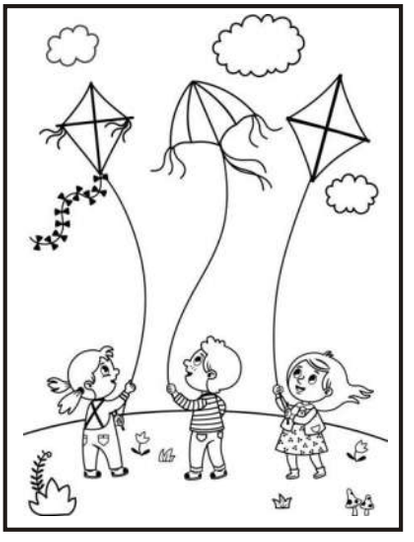
Or

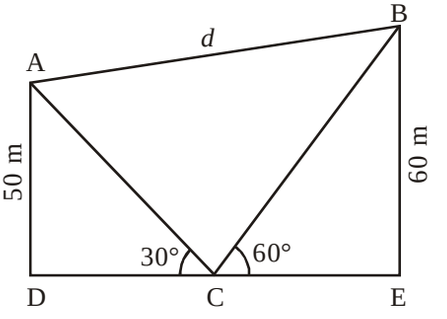
Find the distance between the towns A and B.

38. Kite Festival

Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below, three kites flying together.





In the figure, the angles of elevation of two kites (points A and B) from the hands of a man (point C) are found to be 30° and 60° respectively. Taking AD = 50 m and BE = 60 m.

(*i*) Find the length of string AC.

(*ii*) Find the length of string BC.

(*iii*) Find the distance '*d*' between the two kites.

Or

Find the distance between the points D and E.

Answers

1. (B) 40 **2.** (A) 396 **3.** (A) $m = 12$ and $n = 2$ **4.** (B)

5. (C) Step 4 **6.** (D) Diagonals bisect at right angle and adjacent angles are equal

7. (C) 36 feet **8.** (C) (– 8, 7), (– 8, – 8), (– 8, – 100)

9. (D) 180° – 84° **10.** (B) $\tan\theta = \frac{8}{15}$, $\text{cosec}\,\theta = \frac{17}{8}$, $\sec\theta = \frac{17}{15}$ **11.** (B) 60°

12. (A) $\frac{1}{2}$ **13.** (C) **14.** (C) **15.** (D) ₹ 55

16. (C) 155.33 cm **17.** (D) Mean : 11; Mode : 10.67

18. (A) Empirical probability = $\frac{5}{16}$; Theoretical probability = $\frac{1}{4}$

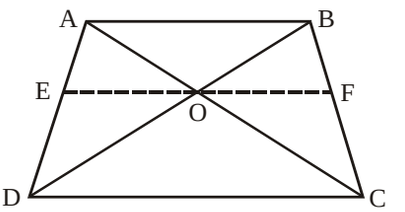
19. (A) Both (A) and (R) are true and (R) is the correct explanation of (A).

20. (B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

21. ***Hint :*** For parallel lines, constant will be different.

(*i*) $2x - 3y - 9 = 0$ (*ii*) $3x - 4y - 10 = 0$

22. In the figure, ABCD is a trapezium with AB || DC.



Construction : Through O, draw EF || DC (See figure).

Now, in ΔADC, EO || DC.

So, by BPT, we get :

$$\frac{AE}{DE} = \frac{AO}{OC} \quad ...(1)$$

Also, since AB || DC and EO || DC, we have :

EO || AB.

So, in ΔDAB, we have :

$$\frac{DE}{AE} = \frac{DO}{BO} \quad \text{(By BPT)}$$

$$\Rightarrow \quad \frac{AE}{DE} = \frac{BO}{DO} \quad ...(2)$$

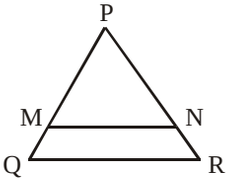
From (1) and (2), we get :

$$\frac{AO}{OC} = \frac{BO}{DO}$$

$$\Rightarrow \quad \frac{AO}{BO} = \frac{OC}{DO} \quad \text{(Proved)}$$

Or

In the figure, PN = 4.8 cm, NR = 1.6 cm, PM = 4.5 cm and MQ = 1.5 cm.



Now, $\frac{PM}{MQ} = \frac{4.5}{1.5} = \frac{3}{1} = 3$

and $\frac{PN}{NR} = \frac{4.8}{1.6} = \frac{3}{1} = 3$.

Thus, $\frac{PM}{MQ} = \frac{PN}{NR}$.

So, by converse of BPT, MN || QR.

23. ***Hint :*** RP shall be calculated by subtracting QR from PQ.

PQ = PT (Tangents from an external point to a circle)

$\Rightarrow$ PQ = 7 cm [$\because$ PT = 7 cm, Given] ...(1)

Also, RQ = RS (Tangents from an external point to a circle)

$\Rightarrow$ RQ = 4 cm [$\because$ SR = 4 cm, given] ...(2)

From (1) and (2), RP = PQ – RQ

= 7 cm – 4 cm = 3 cm.

24. $\sin 60° \cdot \cos 30° - \cos 60° \cdot \sin 30° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$.

25. Area of a sector of circle = $\frac{5}{18}$ × Area of circle.

$\Rightarrow \quad \frac{\pi r^2 \theta}{360°} = \frac{5}{18}\pi r^2.$

$\Rightarrow \quad \frac{\theta}{360°} = \frac{5}{18}.$

$\Rightarrow \quad \theta = \frac{5 \times 360°}{18} = 100°.$

Or

Length of wing = Radius r = 0.98 m.

Angle between two consecutive wings

$$= \theta = \frac{360°}{3} = 120°.$$

So, length of arc described between two consecutive wings

$$= \frac{2\pi r\theta}{360°} = \frac{2 \times 22 \times 0.98 \times 120°}{7 \times 360°} \text{ m}.$$

$$= \frac{6.16}{3} \text{ m} = 2.05 \text{ m}.$$

26. ***Hint :*** By contradiction

Let us suppose that $2\sqrt{3} - 7$ is a rational number. Then, $2\sqrt{3} - 7 = \frac{p}{q}$, where p and q are co-prime and $q \neq 0$.

$\Rightarrow \quad 2\sqrt{3} = \frac{p}{q} + 7 = \frac{p + 7q}{q} \qquad \Rightarrow \quad \sqrt{3} = \frac{p + 7q}{2q}$.

Since p and q are some integers, so, $\frac{p + 3q}{7q}$ is a rational number.

But, we know the fact that $\sqrt{3}$ is an irrational number.

Also, we know that an irrational number cannot be equal to a rational number.

So, equality between $\sqrt{3}$ and $\frac{p + 7q}{2q}$ does not exist.

This contradiction is because of our assumption that $2\sqrt{3} - 7$ is a rational number.

Hence, $2\sqrt{3} - 7$ is an irrational number.

27. α and β are zeroes of the polynomial $6x^2 - 7x - 3$.

So, $$\alpha + \beta = -\frac{(-7)}{6} = \frac{7}{6} \qquad ...(1)$$

and $$\alpha\beta = -\frac{3}{6} = -\frac{1}{2} \qquad ...(2)$$

Now, for the new zeroes, we have :

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7}{6} \times \left(-\frac{2}{1}\right) = -\frac{7}{3} \qquad \text{[From (1) and (2)]}$$

and $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = -\frac{2}{1}$ [From (2)]

$= -2.$

Hence, the new polynomial is

$$x^2 - \left(-\frac{7}{3}\right)x + (-2) = x^2 + \frac{7}{3}x - 2$$

or $3x^2 + 7x - 6$.

28. Let cost of one chair be ₹ x and that of one table be ₹ y.

So, as per given conditions, we have :

$2x + 3y = 5650$ ...(1)

$3x + 2y = 7100$ ...(2)

Multiplying (1) by 3 and (2) by 2, we get

$6x + 9y = 16950$ ...(3)

$6x + 4y = 14200$ ...(4)

Subtracting (4) from (3), we get

$5y = 2750$

$\Rightarrow \quad y = \frac{2750}{5} = 550.$

Putting $y = 550$ in (1), we get

$2x + 3 \times 550 = 5650.$

$\Rightarrow \quad 2x + 1650 = 5650$

$\Rightarrow \quad 2x = 5650 - 1650$

$\Rightarrow \quad 2x = 4000.$

$\Rightarrow \quad x = \frac{4000}{2} = 2000.$

Thus, cost of one chair is ₹ 2000 and that of one table is ₹ 550.

Or

We have :

$9x + 10y = 29$ ...(1)

$10x + 9y = 28$ ...(2)

Multiplying (1) by 9 and (2) by 10, we get

$81x + 90y = 261$ ...(3)

$100x + 90y = 280$ ...(4)

Subtracting (3) from (4), we get

$(100 - 81)x = 280 - 261$

$\Rightarrow \quad 19x = 19 \quad \Rightarrow \quad x = \frac{19}{19} = 1.$

Putting $x = 1$ in (1), we get

$9 \times 1 + 10y = 29$

$\Rightarrow \quad 10y = 29 - 9 = 20.$

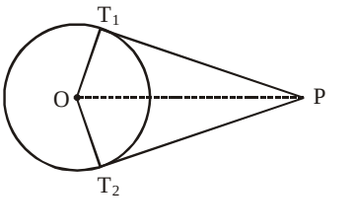
$\Rightarrow \quad y = \frac{20}{10} = 2.$

Thus, $x = 1$ and $y = 2$.

29. Given a circle with centre O. P is a point outside the circle from where two tangents PT_1 and PT_2 have been drawn.

We have to prove that

$PT_1 = PT_2.$



Construction : We join OP.

Now, in ΔOPT_1 and ΔOPT_2,

$OP = OP$ (Common) ...(1)

$OT_1 = OT_2$ (Radii of the same circle) ...(2)

$\angle PT_1O = \angle PT_2O$ (Each 90°) ...(3)

From (1), (2) and (3), by RHS congruency criterion,

$\Delta OPT_1 \cong \Delta OPT_2$

$\Rightarrow$ $PT_1 = PT_2$ (By CPCT)

Hence, proved.

30.
$$\text{LHS} = \frac{\sin x - \tan x}{\sin x . \tan x} = \frac{\sin x - \dfrac{\sin x}{\cos x}}{\sin x \times \dfrac{\sin x}{\cos x}}$$

$$= \frac{\dfrac{\cos x . \sin x - \sin x}{\cos x}}{\dfrac{\sin^2 x}{\cos x}}$$

$$= \frac{\cos x . \sin x - \sin x}{\cos x} \times \frac{\cos x}{\sin^2 x}$$

$$= \frac{\cos x . \sin x - \sin x}{\sin^2 x} = \frac{\sin x(\cos x - 1)}{\sin^2 x}$$

$$= \frac{\cos x - 1}{\sin x} = \text{RHS}.$$

Hence, proved.

Or

$$\text{LHS} = \sec^2\theta(\sec^2\theta - 2) + 1$$
$$= \sec^4\theta - 2\sec^2\theta + 1$$
$$= (\sec^2\theta - 1)^2 = (1 + \tan^2\theta - 1)^2$$
$$= (\tan^2\theta)^2 = \tan^4\theta = \text{RHS}.$$

Hence, proved.

31. (*a*) P(neither ace nor king) $= \dfrac{(52 - 4 - 4)}{52} = \dfrac{44}{52} = \dfrac{11}{13}$.

(*b*) P(non spade) $= \dfrac{(52 - 13)}{52} = \dfrac{39}{52} = \dfrac{3}{4}$.

(*c*) P(a queen of black suit) $= \dfrac{2}{52} = \dfrac{1}{26}$.

32. ***Hint :*** Quadratic equation will be formed, using given condition. Unrealistic solution of the equation will be rejected.

Let the original speed of the plane = x km/hour.

So, time taken to cover 1600 km $= \dfrac{1600}{x}$ hours.

New speed = $(x + 400)$ km/hour.

Time taken to cover 1600 km with new speed $= \dfrac{1600}{x + 400}$ hours.

So, as per given condition,

$$\frac{1600}{x} - \frac{1600}{x + 400} = \frac{40}{60}.$$

$$\Rightarrow \quad 1600\left(\frac{1}{x} - \frac{1}{x+400}\right) = \frac{2}{3}$$

$$\Rightarrow \quad 1600\,\frac{(x+400-x)}{x(x+400)} = \frac{2}{3}$$

$$\Rightarrow \quad 1600 \times 400 \times 3 = 2x(x+400).$$

$$\Rightarrow \quad 2x^2 + 800x - 1600 \times 400 \times 3 = 0.$$

$$\Rightarrow \quad x^2 + 400x - 960000 = 0.$$

$$\Rightarrow \quad x^2 + 1200x - 800x - 960000 = 0$$

$$\Rightarrow \quad x(x+1200) - 800(x+1200) = 0$$

$$\Rightarrow \quad (x+1200)\,(x-800) = 0$$

$$\Rightarrow \quad x = -1200 \quad \text{or} \quad x = 800.$$

So, original speed was 800 km/hour. (Neglecting –1200 as speed)

Or

Hint : Quadratic equation will be formed, using ratio and proportions.

Let the number of students be x.

As per given conditions,

$$\frac{300}{x} - \frac{300}{x+10} = 1.$$

$$\Rightarrow \quad 300\left(\frac{x+10-x}{(x+10)x}\right) = 1$$

$$\Rightarrow \quad 300 \times 10 = x^2 + 10x.$$

$$\Rightarrow \quad x^2 + 10x - 3000 = 0$$

$$\Rightarrow \quad x^2 + 60x - 50x - 3000 = 0$$

$$\Rightarrow \quad x(x+60) - 50(x+60) = 0$$

$$\Rightarrow \quad (x+60)\,(x-50) = 0$$

$$\Rightarrow \quad x = -60 \text{ or } x = 50.$$

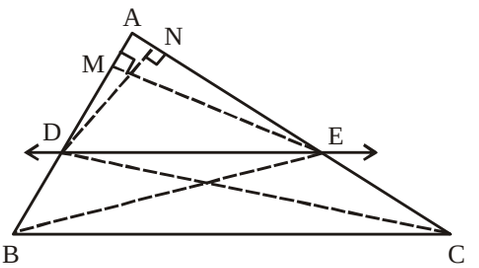
So, neglecting $x = -60$, the number of students is 50.

33. The Basic Proportionality Theorem states that if a line is drawn parallel to a side of a triangle such that it intersects the other two sides at two distinct points, then the other two sides are divided in the same ratio.

Given : A ΔABC in which $DE \parallel BC$, which intersects AB in D and AC in E.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE and CD, draw $EM \perp AB$ and $DN \perp AC$.



Proof : We have :

$$\frac{\text{ar }(\Delta ADE)}{\text{ar }(\Delta DBE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB} \qquad \ldots(1)$$

Similarly, $$\frac{\text{ar }(\Delta ADE)}{\text{ar }(\Delta CDE)} = \frac{\frac{1}{2}\times AE \times DN}{\frac{1}{2}\times EC \times DN} = \frac{AE}{EC} \qquad ...(2)$$

Since ΔDBE and ΔCDE stand on the same base DE and lie between the same parallels DE and BC, so

$$\text{ar }(\Delta DBE) = \text{ar }(\Delta CDE) \qquad ...(3)$$

From (1), (2) and (3), we have :

$$\frac{\text{ar }(\Delta ADE)}{\text{ar }(\Delta DBE)} = \frac{\text{ar }(\Delta ADE)}{\text{ar }(\Delta CDE)}$$

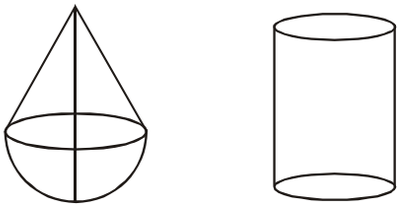
$$\Rightarrow \qquad \frac{AD}{DB} = \frac{AE}{EC} \qquad \text{[From (1) and (2)]}$$

34. ***Hint :*** Volume of water left in the bucket will be calculated by subtracting the volume of the combined solid from the volume of the cylinder.

Volume of the first solid = Volume of cone + Volume of hemisphere

$$= \left(\frac{1}{3}\pi\times\frac{21}{10}\times\frac{21}{10}\times 4+\frac{2}{3}\pi\times\frac{21}{10}\times\frac{21}{10}\times\frac{21}{10}\right) \text{cm}^3.$$

Also, volume of the cylinder = $\pi \times 5 \times 5 \times 9.8\ \text{cm}^3$.



So, volume of the water left in the cylinder

$$= \pi \times 5 \times 5 \times 9.8 - \left(\frac{1}{3}\pi\times\frac{21\times21\times4}{10\times10}+\frac{2}{3}\pi\times\frac{21\times21\times21}{10\times10\times10}\right)$$

$$= \left\{\frac{22}{7}\times 25\times\frac{98}{10}-\left(\frac{1}{3}\times\frac{22}{7}\times\frac{21\times21\times4}{100}+\frac{2}{3}\times\frac{22}{7}\times\frac{21\times441}{1000}\right)\right\}\text{cm}^3.$$

$$= \left\{11\times5\times14-\left(\frac{22\times21}{25}\right)-\left(\frac{44\times441}{1000}\right)\right\}\text{cm}^3.$$

$$= 77\left\{10-\frac{6}{25}-\frac{4\times63}{1000}\right\}\text{cm}^3.$$

$$= 77\left\{\frac{10000-240-252}{1000}\right\}\text{cm}^3$$

$$= \frac{77\times9508}{1000}\text{cm}^3$$

$$= 732.116\ \text{cm}^3.$$

Or

Hint : Volume of the wood used will be found by subtracting volumes of 4 conical depressions and one cubical depression from the volume of the cuboid.

Volume of the wood used = Volume of cuboid – 4 × Volume of a cone – Volume of a cube.

$$= \left\{20 \times 15 \times 10 - 4 \times \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 2.1 - 9 \times 9 \times 9\right\} \text{ cm}^3.$$

$$= \left(3000 - 4 \times \frac{22}{21} \times \frac{25}{100} \times \frac{21}{10} - 729\right) \text{ cm}^3$$

$$= \left(3000 - \frac{11}{5} - 729\right) \text{ cm}^3.$$

$$= \left(\frac{15000 - 11 - 3645}{5}\right) \text{ cm}^3.$$

$$= \left(\frac{15000 - 3656}{5}\right) \text{ cm}^3 = \frac{11344}{5} \text{ cm}^3.$$

35.

Literacy rate (in %) (Class interval)	No. of cities (Frequency)	Cumulative frequency
20 – 30	3	3
30 – 40	2	5
40 – 50	6	11
50 – 60	15	26
60 – 70	8	34
70 – 80	7	41
80 – 90	5	46
90 – 100	4	50
Total	50	

The modal class is 50-60.

Here, $l = 50$, $f_1 = 15$, $f_0 = 6$, $f_2 = 8$ and $h = 10$.

Now, $$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h.$$

$\Rightarrow$ $$\text{Mode} = 50 + \left(\frac{15 - 6}{2 \times 15 - 6 - 8}\right) \times 10$$

$$= 50 + \frac{9 \times 10}{16} = 50 + \frac{45}{8}$$

$$= 50 + 5.625 = 55.625.$$

Now, $$\text{median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h.$$

The median class is 50 – 60, since

$$\frac{N}{2} = \frac{50}{2} = 25.$$

Here, $l = 50$, N = 50, $cf = 11$, $f = 15$ and $h = 10$.

$$\Rightarrow \quad \text{Median} = 50 + \frac{\left(\frac{50}{2} - 11\right)}{15} \times 10$$

$$= 50 + \frac{14 \times 10}{15} = 50 + \frac{28}{3}$$

$$= 50 + 9.33 = 59.33 \text{ (approx.)}.$$

36. (*i*) We have : $S_n = 0.1n^2 + 7.9n$.

Putting $n = 1$, we have $S_1 = 0.1 \times 1^2 + 7.9 \times 1 = 8$.

So, first term is 8 and hence diameter of the core is 8.

(*ii*) Putting $n = 2$, we get $S_2 = 0.1 \times 4 + 7.9 \times 2 = 16.2$.

So, second term of AP = 16.2 – 8 = 8.2

Hence, common difference = 8.2 – 8 = 0.2

So, thickness of the paper = $\frac{1}{2} \times 0.2 = 0.1$

(*iii*) $S_{n-1} = 0.1\,(n-1)^2 + 7.9\,(n-1) = 0.1\,(n^2 - 2n + 1) + 7.9n - 7.9$

$= 0.1n^2 - 0.2n + 0.1 + 7.9n - 7.9$

$= 0.1n^2 + 7.7n - 6.9$

Or

$S_n - S_{n-1} = (0.1n^2 + 7.9n) - (0.1n^2 + 7.7n - 6.9)$

$= 0.1n^2 + 7.9n - 0.1n^2 + 7.7n + 6.9$

$= -15.6n + 6.9$

37. (*i*) Meena hometown (5, 2).

(*ii*) Anjali hometown (1, 8).

(*iii*) Coordinates of D are (– 4, 4). So, distance travelled by Meena to her hometown

$\sqrt{(5+4)^2 + (2-4)^2} = \sqrt{81+4} = \sqrt{85}$ units.

Or

Distance between towns A and B = $\sqrt{(5-1)^2 + (2-8)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$ units.

38. (*i*) From ΔADC, $\frac{AD}{AC} = \sin 30° \Rightarrow \frac{50}{AC} = \frac{1}{2} \Rightarrow AC = 50 \times 2 = 100$ m.

(*ii*) From ΔBEC, $\frac{BE}{BC} = \sin 60° \Rightarrow \frac{\sqrt{3}}{2} \Rightarrow \frac{60}{BC} = \frac{\sqrt{3}}{2} \Rightarrow BC = \frac{120}{\sqrt{3}} = 40\sqrt{3}$ m.

(*iii*) $\angle ACB = 180° - 30° - 60° = 90°$.

So, $AB^2 = AC^2 + BC^2 \Rightarrow d^2 = 100^2 + (40\sqrt{3})^2$

$\Rightarrow d^2 = 10000 + 4800 = 14800 \Rightarrow d = \sqrt{14800}$ m $= 20\sqrt{37}$ m.

Or

$\frac{50}{DC} = \cos 30° \Rightarrow \frac{50}{DC} = \frac{\sqrt{3}}{2} \Rightarrow DC = \frac{100}{\sqrt{3}}$ m.

Also, $\frac{60}{CE} = \cos 60° \Rightarrow \frac{60}{CE} = \frac{1}{2} \Rightarrow CE = 120$ m.

So, distance between D and E $= \left(120 + \frac{100}{\sqrt{3}}\right)$ m $= \left(\frac{120\sqrt{3} + 100}{\sqrt{3}}\right)$ m.

10 SAMPLE QUESTION PAPER

MATHEMATICS (Basic)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions : Same as in CBSE Sample Question Paper.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. Given that HCF (26, 91) = 13, then LCM of (26, 91) is :

(A) 2366 (B) 182 (C) 91 (D) 364

2. $n^2 - 1$ is divisible by 8, if n is :

(A) an integer (B) a natural number (C) an odd integer (D) an even integer

3. Which of these is a quadratic equation?

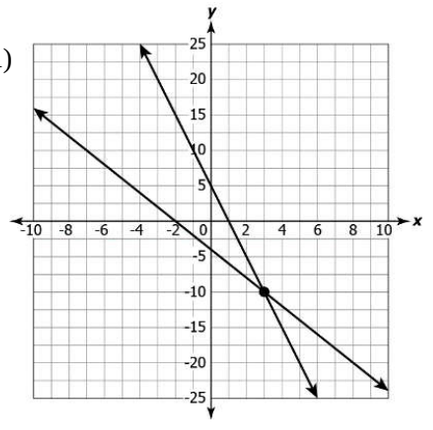
(A) $x^2 - x = x^2 + 2$ (B) $x^2 + \frac{1}{x^2} + 1 = 0$

(C) $(x + 1)(x + 3) = (x - 1)(x - 4)$ (D) $x(x + 1) = 2x - 3$

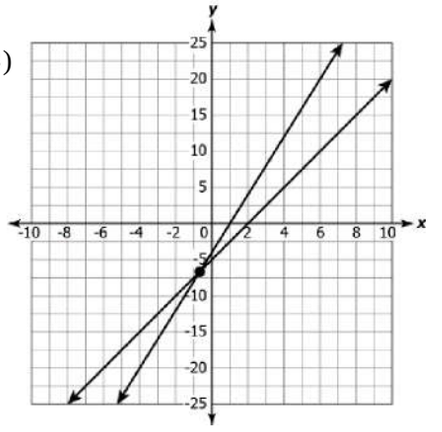
4. Arnav wants to plant some saplings in columns. If he increases the number of saplings in a column by 4, the number of columns decreases by 1. If he decreases the number of saplings by 5 in a column, the number of columns increased by 2.

Which of these graphs relates the number, x, of columns and the number, y, of plants in a column?

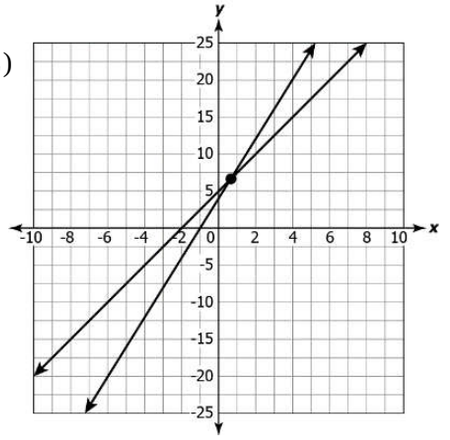
(A)



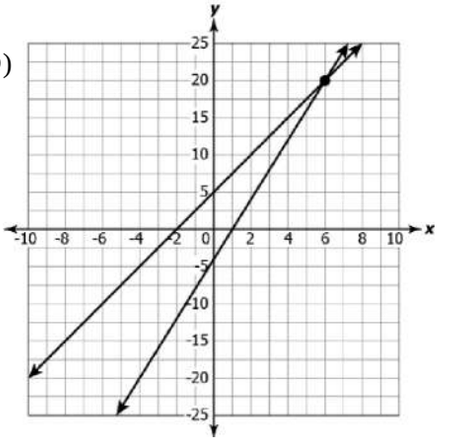
(B)



(C)

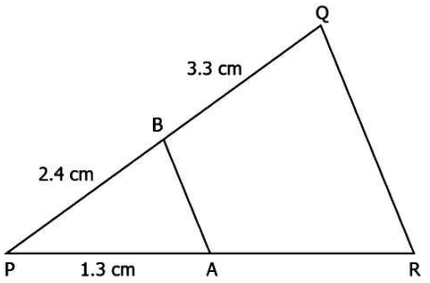


(D)



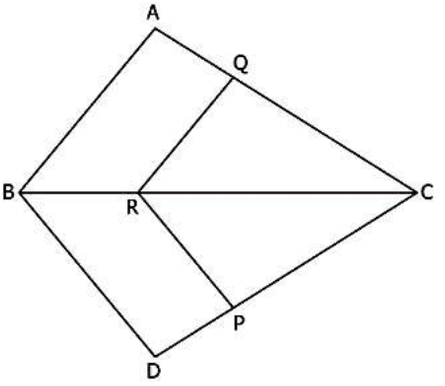
5. Consider the equation $kx^2 + 2x = c\,(2x^2 + b)$
For the equation to be quadratic, which of these cannot be the value of k?
(A) c (B) $2c$ (C) $3c$ (D) $2c + 2b$

6. In the figure below, BA || QR.



To the nearest tenth, what is the length of AR?
(A) 1.8 cm (B) 1.7 cm (C) 1.4 cm (D) 2.2 cm

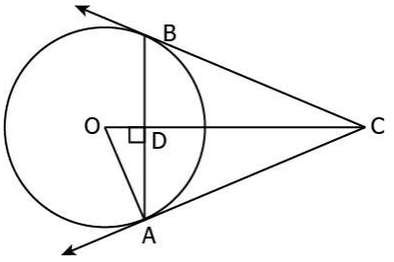
7. In the given figure, QR || AB, RP || BD, CQ = $x + 2$, QA = x, CP = $5x + 4$, PD = $3x$.



The value of x is _____.
(A) 1 (B) 3 (C) 6 (D) 9

8. On a graph, two-line segments, AB and CD of equal length are drawn. Which of these could be the coordinates of the points, A, B, C and D?
(A) A(– 3, 4) B(– 1, 2) and C(3, 4) D(1, 2) (B) A(– 3, – 4) B(– 1, 2) and C(3, 4) D(1, 2)
(C) A(– 3, 4) B(– 1, – 2) and C(3, 4) D(1, 2) (D) A(3, 4) B(– 1, 2) and C(3, 4) D(1, 2)

9. In figure is shown, AC and BC are two tangents to the circle O with radius 5 cm.



If OC = 13 cm and OD : DC = 3 : 10, then what is the perimeter of triangle ABC ?
(A) 24 cm (B) 30 cm (C) 32 cm (D) 34 cm

10. If $\sin\theta = \frac{7}{\sqrt{85}}$, what are the values of $\tan\theta$, $\cos\theta$ and $\text{cosec}\,\theta$?

(A) $\tan\theta = \frac{7}{6}, \cos\theta = \frac{6}{\sqrt{85}}$ and $\text{cosec}\,\theta = \frac{\sqrt{85}}{7}$ (B) $\tan\theta = \frac{6}{7}, \cos\theta = \frac{7}{\sqrt{85}}$ and $\text{cosec}\,\theta = \frac{\sqrt{85}}{7}$

(C) $\tan\theta = \frac{7}{6}$, $\cos\theta = \frac{7}{\sqrt{85}}$ and $\text{cosec}\,\theta = \frac{\sqrt{85}}{7}$ (D) $\tan\theta = \frac{6}{7}$, $\cos\theta = \frac{6}{\sqrt{85}}$ and $\text{cosec}\,\theta = \frac{\sqrt{85}}{6}$

11. If $2\sin 2\theta = \sqrt{3}$, then the value of θ is :

(A) 90° (B) 30° (C) 45° (D) 60°

12. The two legs AB and BC of right triangle ABC are in a ratio 1:3. What will be the value of sin C ?

(A) $\frac{1}{\sqrt{10}}$ (B) $\frac{3}{\sqrt{10}}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

13. To form a circle of radius r, four minor sectors of equal measure are joined.

Which of these options completes the sentence below?

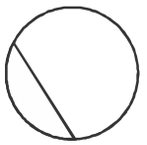
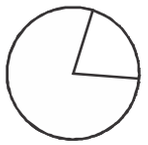
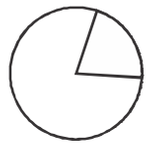
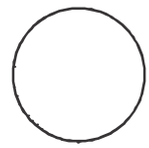
The sum of the area of the four minor sectors is equal to the _____.

(A) circumference of the circle of radius r. (B) area of the semicircle of diameter $2r$.

(C) area of the circle of diameter $2r$. (D) circumference of the circle of diameter r.

14. To show the minor segment of a circle, a student shades the region enclosed between a chord and the minor arc.

Which of these shows the region the student could have shaded?

(A)  (B)  (C)  (D) 

15. The radius of the largest right circular cone that can be cut out from a cube of edge 6.4 cm is

(A) 6.4 cm (B) 12.8 cm (C) 3.2 cm (D) 1.6 cm

16. Amith grows cucumbers in his farm. He collects some of them and measures their lengths and represents his data as shown.

Length (in mm)	Number of cucumbers
110-120	10
120-130	18
130-140	12
140-150	n
150-160	26
160-170	11
170-180	19

When calculated using assumed mean method, Amith gets the mean length of the cucumber as 147.25 cm. Which of the following statement is true?

(A) There are a smaller number of cucumbers of length (140-150) mm than of length (120-130) mm.

(B) There are a smaller number of cucumbers of length (140-150) mm than of length (170-180) mm.

(C) There are a greater number of cucumbers of length (140-150) mm than of length (150-160) mm.

(D) There are a greater number of cucumbers of length (140-150) mm than of length (130-140) mm.

17. The table below shows the time taken by a group of 40 students to solve a work sheet.

Number of minutes	Number of students
20 – 24	1
24 – 28	2
28 – 32	4
32 – 36	k
36 – 40	10
40 – 44	8
44 – 48	3

Which of the following statements is true?

(A) The mean time taken by the students is more than the sum of time taken by all the students to solve the worksheet.

(B) The time taken by the maximum number of students is less than the mean time taken by the students to solve the worksheet.

(C) The time taken by the maximum number of students is more than the mean time taken by the students to solve the worksheet.

(D) The time taken by the maximum number of students is more than the sum of time taken by all the students to solve the worksheet.

18. Mandy has a bag containing 1 red, 1 green, 1 yellow, 1 black and 1 blue ball. She randomly picks the ball from the bag notes it colour and keeps it back in the bag. She repeats this 40 times. The table shows the number of times each colour ball she gets. The number of times the black ball is picked is missing in the table.

Red ball	Green ball	Yellow ball	Black ball	Blue ball
10	6	5	?	10

She then repeats the experiment 10 more times and gets red ball twice, green ball once, yellow ball thrice, black ball once and blue ball thrice.

Which of these is a valid conclusion as the number of trials of the experiment increases?

(A) The empirical probability of picking yellow ball gets closer to its theoretical probability.

(B) The empirical probability of picking red ball does not get closer to its theoretical probability.

(C) The empirical probability of picking yellow ball gets further away from its theoretical probability.

(D) The empirical probability of picking red ball becomes equal to its theoretical probability.

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. **(A) :** If HCF(a, 8) = 4 and LCM(a, 8) = 24, then a is equal to 12.

(R) : Product of two numbers is equal to product of their HCF and LCM.

20. **(A) :** Centroid of the triangle with vertices (2, 3), (3, 4) and (–1, 2) is $\left(\frac{4}{3}, \frac{5}{3}\right)$.

(R) : Centroid of the triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

SECTION – B

Section-B consists of 5 questions of 2 marks each.

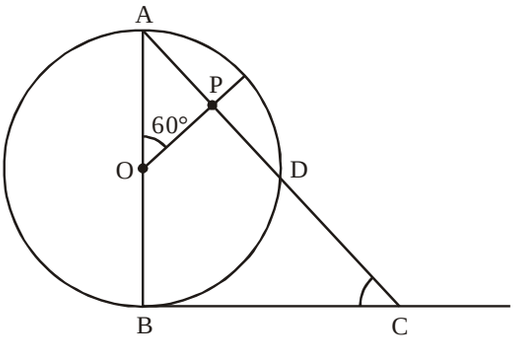
21. Given the linear equation $x - 2y - 6 = 0$, write another linear equation in these two variables, such that the geometrical representation of the pair so formed is :
(*i*) coincident lines, (*ii*) intersecting lines.

22. Prove that the diagonals of a trapezium intersect each other in the same ratio.

Or

If two triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of corresponding angle bisector segments.

23. In the figure, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.



24. If it is given that $3 \operatorname{cosec} \theta = 4$, then verify that $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$.

25. The length of an arc of a circle of radius 18 cm is 15p cm. Find the central angle of this arc.

Or

For an advertisement purpose, a mall in the city is spreading a white light in the night time over a sector of angle 70° to a distance of 10 km. Find the area of the city over which the light is spread.

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Explain whether the number $3 \times 5 \times 13 \times 46 + 23$ is a prime number or a composite number.

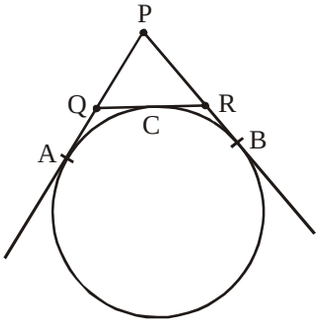
27. Find a quadratic polynomial whose zeroes are $5 + \sqrt{3}$ and $5 - \sqrt{3}$.

28. Draw the graph of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and x-axis. Find the area of the shaded region.

Or

Solve for x and y : $7x - 5y = 2$
$x + 2y = 3$

29. In the figure, perimeter of DPQR is 20 cm. Find the length of tangent PA.



30. If $3\cos\theta = 1$, find the value of $\dfrac{6\sin^2\theta + \tan^2\theta}{4\cos\theta}$.

Or

If $\sin\theta = \dfrac{3}{5}$, evaluate $\dfrac{\text{cosec}\,\theta - \cot\theta}{2\cot\theta}$.

31. One card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of getting

(A) a king of red colour. (B) a face card.

(C) a red face card. (D) the jack of hearts.

SECTION – D

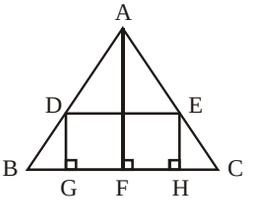
Section-D consists of 4 questions of 5 marks each.

32. Solve the quadratic equation $3x^2 - 3\sqrt{21}x + 14 = 0$.

Or

The sum of ages of father and his son is 45 years. 5 years ago, the product of their ages (in years) was 124. Determine their present ages.

33. In the given figure, ABC is a triangle and GHED is a rectangle. BC = 12 cm, HE = 6 cm, FC = BF and altitude AF = 24 cm. Find the area of the rectangle.



34. From a solid cylinder of the height of 30 cm and radius of 7 cm, a conical cavity of height of 24 cm and same radius is hollowed out. Find the total surface area of the remaining solid.

Or

From a solid cylinder, whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

35. Following are the ages of asthmatic patients admitted during a year in a hospital. Find the mean age of the patients.

Age (in years)	0-8	8-16	16-24	24-32	32-40	40-48	48-56	56-64
Number of patients	6	25	12	13	11	14	11	8

SECTION – E

Case study based questions are compulsory.

36. Anita's mother start a new shoe shop. To display the shoes, she put 3 pairs of shoes in 1st row, 5 pairs in 2nd row, 7 pairs in 3rd row and so on.

On the basis of above information, answer the following questions.

(*i*) How many pairs of shoes are there in the 5th row ?

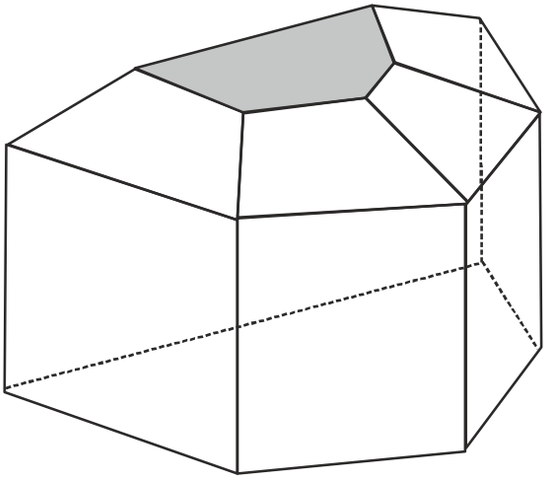
(*ii*) How many pairs of shoes are there in the 7th row ?

(*iii*) If she puts a total of 120 pairs of shoes, then find the number of rows required.

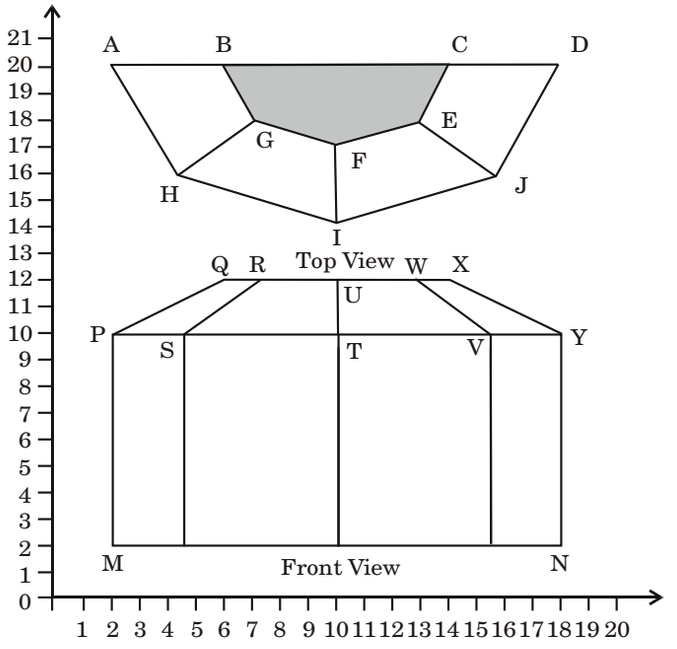
Or

What is the difference of pairs of shoes in 17th row and 10th row ?

37. A sunroom is to be built on the wall of a house. The four walls of sunroom will be square transparent glass panels. The roof of the sunroom will be made up of four transparent glass panels in the shape of congruent trapeziums and one tinted glass panel in the shape of a semi-regular octagon as shown in the figure.



Top and front view of this sunroom are sketched in the coordinate plane as shown below. On the basis of this information, answer the following questions :



(*i*) What is the mid-point of the line-segment BC ?

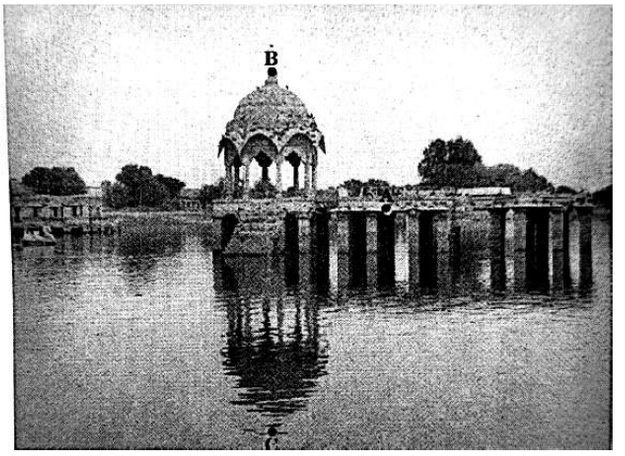
(*ii*) What is the mid-point of the line-segment FG ?

(*iii*) Find the coordinates of the point, which divides the line segment VW in the ratio 1 : 3.

Or

Find the coordinates of the point equidistant from points G and E.

38. Gadisagar lake is located in the Jaisalmer district of Rajasthan. It was built by king of Jaisalmer and rebuilt by Gadsi Singh in 14th Century. The lake has many *chattris*. One of them is shown below :



Observe the picture. From a point A, which is h metre above from water level, the angle of elevation of top of *chattari* (point B) is 45° and angle of depression of its reflection in water (point C) is 60°. If the height of chhattri above the water level is 10 m, then

(*i*) Draw a well-labelled figure based on the above information.

(*ii*) Find the height (h) of the point A above the water level.

(*iii*) What is the perpendicular distance of point A from the *chattris* ?

Or

Find the value of AB^2 (in m^2).

11 SAMPLE QUESTION PAPER

MATHEMATICS (Basic)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions : Same as in CBSE Sample Question Paper.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. If two positive integers a and b are written as $a = x^2y^2$ and $b = xy^2$; x, y are prime numbers, then HCF (a, b) is :

(A) xy (B) xy^2 (C) x^2y^3 (D) x^2y^2

2. The product of the HCF and LCM of the smallest prime number and the smallest composite number is :

(A) 2 (B) 4 (C) 6 (D) 8

3. A quadratic equation can have :

(A) at least two roots (B) at most two roots (C) always two roots (D) only one root

4. Rohit earned ₹ 3550 by selling some bags each for ₹ 500 and some baskets each for ₹ 150. Aarav earned ₹ 3400 by selling the same number of bags each for ₹ 400 and the same number of baskets each for ₹ 200 as Rohit sold. Which of these equations relates the number of bags x, and the number of baskets, y?

(A) $500x + 150y = 3550$ and $400x + 200y = 3400$

(B) $500x + 150y = 3400$ and $400x + 200y = 3550$

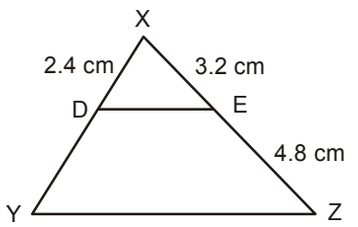
(C) $400x + 150y = 3550$ and $500x + 200y = 3400$

(D) $500x + 200y = 3550$ and $400x + 150y = 3400$

5. The roots of $ax^2 + bx + c = 0$, $a \neq 0$ are real and unequal. Which of these is true about the value of discriminant, D?

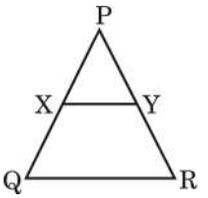
(A) $D < 0$ (B) $D > 0$ (C) $D = 0$ (D) $D \leq 0$

6. In the given figure, in ΔXYZ, DE || YZ, so that the lengths of sides XD, XE and EZ (in centimeters) are 2.4, 3.2 and 4.8 respectively. Then the length of XY (in centimeters) is :



(A) 3.6 (B) 6 (C) 6.4 (d) 1.6

7. If in ΔPQR, $XY \parallel QR$, $PX = x - 2$, $XQ = 3x$, $PY = x + 2$ and $YR = 9x$, then the value of x is :



(A) 1 (B) 2 (C) 3 (D) 4

8. The coordinates of the point where the line $x - y = 5$ cuts y-axis are :

(A) $(0, -5)$ (B) $(5, 0)$ (C) $(0, 5)$ (D) $(-5, 0)$

9. To a circle with centre O and radius 6 cm, a tangent from which of these points is not possible, if the distance of the point from centre is :

(A) 8 cm (B) 10 cm (C) 12 cm (D) 4 cm

10. If $\tan A = \frac{5}{12}$, then the value of $(\sin A + \cos A) \times \sec A$ is :

(A) $\frac{6}{13}$ (B) $\frac{7}{12}$ (C) $\frac{17}{12}$ (D) $\frac{12}{17}$

11. If $3 \cos \theta = 2 \sin \theta$, then the value of $\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$ is :

(A) $\frac{1}{8}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

12. Given that $\cos \theta = \frac{m}{n}$, then $\tan \theta$ is equal to :

(A) $\frac{n}{\sqrt{n^2 - m^2}}$ (B) $\frac{\sqrt{n^2 - m^2}}{m}$ (C) $\frac{\sqrt{n^2 - m^2}}{n}$ (D) $\frac{n}{m}$

13. Area of the sector of a circle of radius $2r$ and central angle x^o is :

(A) $\frac{\pi x^2}{360}$ (B) $\frac{\pi x r^2}{90}$ (C) $\frac{\pi r^2 x}{1440}$ (D) $\frac{\pi r x}{720}$

14. If the area of a sector of a circle is $\frac{5}{9}$ of area of the circle, then its central angle is :

(A) 100^o (B) 150^o (C) 200^o (D) 210^o

15. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is :

(A) $60\ \pi\ cm^2$ (B) $68\ \pi\ cm^2$ (C) $120\ \pi\ cm^2$ (D) $136\ \pi\ cm^2$

16. Construction of a cumulative frequency table is useful in determining the :

(A) mean (B) median

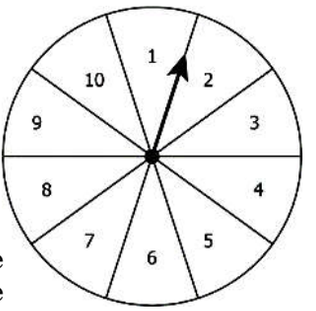
(C) mode (D) all the above three measures

17. The empirical relation between mean, mode and median is

(A) 3 Median – 2 Mean = Mode (B) 2 Median – 3 Mean = Mode

(C) Mode – 2 Mean = 3 Median (D) 2 Mean + Median = 3 Mode

18. A spinner is shown below.



Some of the events are listed below, when the spinner is spinned.

Event A : The spinner lands on a multiple of 11.

Event B: The spinner lands on a number less than 11.

Event C: The spinner lands on a number more than 10.

Which of the following statement is true about the three events?

(A) Probability of Event A is 1, so A is a sure event while the probabilities of Events B and C are 0, so they are impossible events.

(B) Probability of Event A is 1, so A is an impossible event while the probabilities of Events B and C are 0, so they are sure events.

(C) Probability of Event B is 1, so B is a sure event while the probabilities of Events A and C are 0, so they are impossible events.

(D) Probability of Event B is 1, so B is an impossible event while the probabilities of Events A and C are 0, so they are sure events.

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. **(R) :** $5+\sqrt{3}$ is an irrational number.

(R) : Sum of a rational number and an irrational number is an irrational number.

20. **(A) :** The point P lying on the y-axis and equidistant from the points (2, 3) and (– 4, 1) is (0, –1).

(R) : Point P has coordinates (0, y) and $PA^2 = PB^2$ and so $(0 - 2)^2 + (y - 3)^3 = (0 + 4)^2 + (y - 1)^2$.

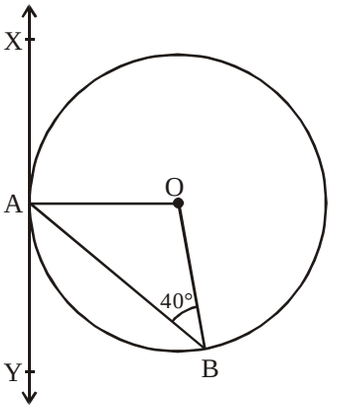
SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. Given the linear equation $3x - 4y - 7 = 0$, write another linear equation in these two variables such that the geometrical representation of the pair so formed is :

(i) intersecting lines, (ii) parallel lines.

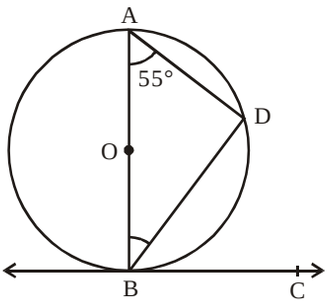
22. In the figure, XAY is a tangent to the circle centered at O. If $\angle ABO = 40°$, then find $m\angle BAY$ and $m\angle AOB$.



Or

Two concentric circles are of radii 4 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

23. In the figure, BC is tangent to the circle at point B of circle centred at O. BD is a chord of the circle so that $\Delta BAD = 55°$. Find $m\Delta DBC$.



24. Prove the following identity : $\left(1+\frac{1}{\tan^2 A}\right).\left(1+\frac{1}{\cot^2 A}\right)=\left(\frac{1}{\cos^2 A-\cos^4 A}\right)$

25. 20 cm long cylindrical pencils of diameter 1 cm each are packed in a cuboidal box in two rows. If there are 20 pencils to be arranged in this box, then what is the surface area of this box ?

Or

The radius and slant height of a right circular cone are in the ratio of 7 : 13 and its curved surface area is 286 cm^2. Find its radius of the cone. $\left(\text{Use } \pi=\frac{22}{7}\right)$

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.

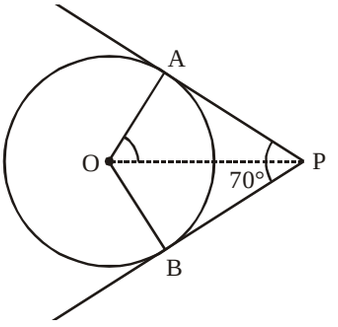
27. Find a quadratic polynomial, the sum and product of whose zeroes are – 8 and 12 respectively. Hence, find the zeroes.

28. The difference of two numbers is 66. If one number is four times the other, find the numbers.

Or

Solve by substitution : $x - 4y = 1$, $2x - 5y = 5$

29. In the figure, if tangents PA and PB drawn from a point P to a circle with centre O, are inclined to each other at an angle of 70°, then find the measure of $\angle POA$.



30. Prove the following identity : $(\text{cosec}\ \theta - \sin\theta).(\sec\theta - \cos\theta)$.

Or

Prove the identity : $\frac{1+\cot^2\theta}{1-\cos^2\theta} - \text{cosec}^4\ \theta = 0$.

31. Two fair dice are rolled simultaneously. What is the probability that the sum of the numbers obtained is a multiple of 3 ?

SECTION – D

Section-D consists of 4 questions of 5 marks each.

32. Using quadratic formula, solve the following equation : $abx^2 + (b^2 - ac)x - bc = 0$

Or

A faster train takes one hour less than a slower train for a journey of 200 km. If the speed of slower train is 10 km/h less than that of faster train, find the speeds of two trains.

33. If $\Delta ABC \sim \Delta DEF$ and AX, DY are respectively the medians of ΔABC and ΔDEF. Then prove that :

(*i*) $\Delta ABX \sim \Delta DEY$ (*ii*) $\Delta ACX \sim \Delta DFY$ (*iii*) $\frac{AX}{DY} = \frac{BC}{EF}$.

34. A spherical glass vessel has a cylindrical neck 8 cm long and 1 cm in radius. The radius of the spherical part is 9 cm. Find the amount of water (in litres) it can hold, when filled completely.

Or

The dimensions of a metallic cuboid are 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find the surface area of the cube.

35. In an office, transport expenditures of 80 employees are given below :

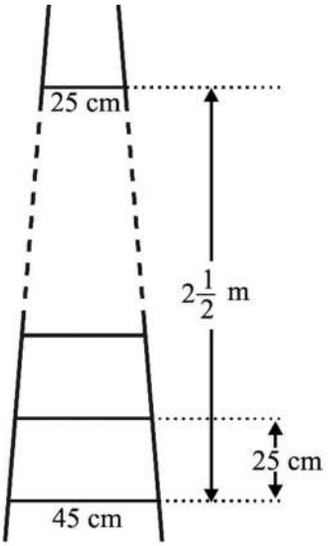
Expenditure on transport (in ₹)	No. of employees
0 – 200	14
200 – 400	19
400 – 600	15
600 – 800	11
800 – 1000	20
1000 – 1200	11

Find the modal expenditure of transport.

SECTION – E

Case study based questions are compulsory.

36. A ladder has rungs 25 cm apart. (see the figure).



The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. The top and the bottom rungs are 2½ m apart.

(*i*) Find the number of the rungs.

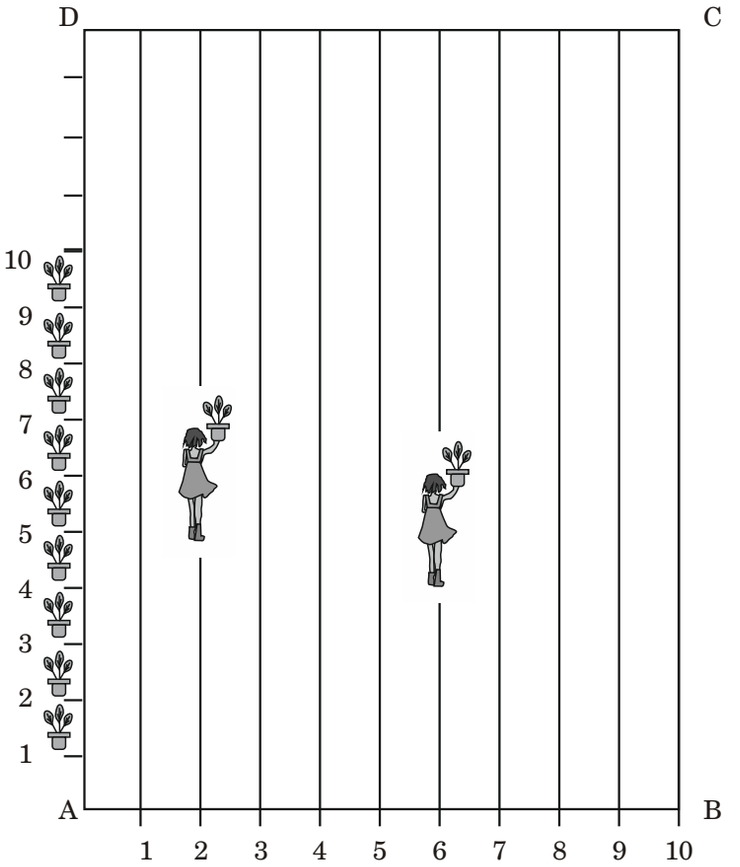
(*ii*) Find the uniform decrease of length between two consecutive rungs.

(*iii*) Find the length of the 4th rung from the bottom.

Or

What is the length of the wood required for the rungs ?

37. To conduct sports activities on the annual sports meet of a girl school, lines were drawn on the rectangular ground ABCD by chalk powder at a distance of 1 m each and 100 flower pots were placed at a distance of 1 m from each other alongside AD as shown in the figure.



A student Neetu runs $\frac{1}{5}$th of distance AD on the second line and posts a green flag. Another student Mary runs $\frac{1}{4}$th of the distance AD on the 6th line and posts a red flag.

(*i*) At how much distance Neetu posted the green flag from the starting point of the second line?

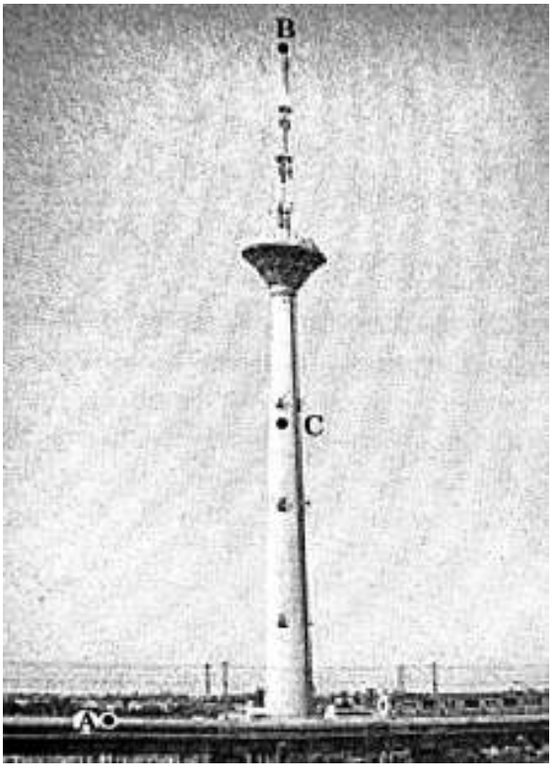
(*ii*) At how much distance Mary posted the red flag from the starting point of the sixth line?

(*iii*) Find the distance between the green and red flags.

Or

If a third student Namita posts a white flag at the mid-point of line segment joining green and red flags, then find the coordinates of the mid-point for the white flag.

38. The following TV Tower was built in 1988 and is located in Pitampura, Delhi. It has an observation deck. Observe the picture given below :



The TV Tower stands vertically on the ground. From a point 'A' on the ground, the angle of elevation of top of the tower (point 'B') is 60°. There is a point 'C' on the tower which is 78 m (approx.) above the ground. The angle of elevation of the point C from point A is found to be 30°.

(*i*) Draw a well-labelled figure, based on the information given above.

(*ii*) Find the distance of the tower from point A.

(*iii*) Find the height of the tower.

Or

What is the distance of point C from B ?

12 SAMPLE QUESTION PAPER

MATHEMATICS (Basic)

Time Allowed : 3 Hours **CLASS–X** *Maximum Marks* : 80

General Instructions : Same as in CBSE Sample Question Paper.

SECTION – A

Section-A consists of 20 questions of 1 mark each.

1. Pairs of natural numbers whose least common multiple is 78 and the greatest common divisor is 13 are :

(A) 58 and 13 or 16 and 29 (B) 68 and 23 or 36 and 49

(C) 18 and 73 or 56 and 93 (D) 78 and 13 or 26 and 39

2. If HCF (72, 120) = 24, then LCM (72, 120) is

(A) 240 (B) 360 (C) 1728 (D) 2880

3. Which of the following is a quadratic equation ?

(A) $(x + 3)(x - 1) = x^2 - 4x + 9$ (B) $(x - 3)(2x + 1) = x(x + 5)$

(C) $x^2 + 3x + 1 = (x - 2)^2$ (D) $(x + 1)^2 - x(x - 3) = 0$

4. The lines representing the linear equations $2x - y = 3$ and $4x - y = 5$.

(A) intersect at a point (B) are parallel

(C) are coincident (D) intersect at exactly two points

5. The equation $2x^3 + 9x^2 - 9x - 2(2x^2 + 1) = 0$ is a

(A) linear equation (B) quadratic equation

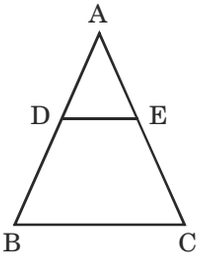
(C) biquadratic equation (D) cubic equation

6. The height of mountains is found out using the idea of indirect measurements which is based on the

(A) Principle of congruent figures (B) Principle of similarity of figures

(C) Principle of equality of figures (D) Principle of non-similarity of figures

7. In the given figure $\frac{AD}{BD} = \frac{AE}{EC}$ and $\angle ADE = 70^\circ$, $\angle BAC = 50^\circ$, then $\angle BCA$ is equal to.



(A) 70º (B) 50º (C) 80º (D) 60º

8. The distance between two points, M and N, on a graph is given as $\sqrt{10^2 + 7^2}$. The coordinates of point M are (– 4, 3). Given that the point N lies in the first quadrant, which of the following is true about the all possible x coordinates of point N?

(A) They are multiples of 2. (B) They are multiples of 3.
(C) They are multiples of 5. (D) They are multiples of 6.

9. If tangents PA and PB from a point to a circle with centre O are inclined to each other at an angle of 50°, then ∠AOB is equal to
(A) 50° (B) 100° (C) 65° (D) 130°

10. Which of these is equivalent to $\frac{2\tan x(\sec^2 x - 1)}{\cos^3 x}$?
(A) $2\tan^3 x \operatorname{cosec} x$ (B) $2\tan^3 x \sec^3 x$ (C) $2\tan^3 x \operatorname{cosec}^3 x$ (D) $2\cot^3 x \sec^3 x$

11. Which of the following option makes the statement below true?

$$\frac{\frac{1}{\sec x} + \sec x}{\cos^2 x - 1 - \tan^2 x} = ?$$

(A) $-\operatorname{cosec} x \cot x$ (B) $-\sec x \cot x$ (C) $-\operatorname{cosec} x \tan x$ (D) $-\sec x \tan x$

12. $[\cos^4 A - \sin^4 A]$ is equal to :
(A) $2\cos^2 A + 1$ (B) $2\cos^2 A - 1$ (C) $2\sin^2 A - 1$ (D) $2\sin^2 A + 1$

13. The perimeter of a quadrant of a circle of radius 'r' is :
(A) $\frac{\pi r}{2}$ (B) $2\pi r$ (C) $\frac{r}{2}[\pi + 4]$ (D) $2\pi r + \frac{r}{2}$

14. A solid sphere of radius r is divided into two equal halves. The total surface area of the two parts is
(A) $3\pi r^2$ (B) $4\pi r^2$ (C) $6\pi r^2$ (D) $8\pi r^2$

15. The curved surface area of a right circular cone of height 7 cm and diameter 48 cm is
(A) 150π cm^2 (B) 600π cm^2 (C) 300π cm^2 (D) 350π cm^2

16. A grouped data is shown below

Class Interval	Frequency	Cumulative frequency
0-10	4	4
10-20	a	$4 + a$
20-30	4	$8 + a$
30-40	b	$8 + a + b$
40-50	5	$13 + a + b$

If the median of the grouped data is 22.50 and the total frequency is 20, then what is the value of a ?
(A) 1 (B) 2 (C) 4 (D) 5

17. The table below shows the time spent by 100 students on exercising every day.

Time (in minutes)	Number of students
0-10	x
10-20	24
20-30	32
30-40	21
40-50	y
50-60	5

If the median time spent by the students on exercising is 26.25 minutes which statement correctly compares the frequencies of the class intervals 0-10 and 40-50 ?
(A) There are twice as many students who exercise 0-10 minutes each day as the number of students who exercise 40-50 minutes each day.

(B) There are twice as many students who exercise 40-50 minutes each day as the number of students who exercise 0-10 minutes each day.

(C) There are thrice as many students who exercise 0-10 minutes each day as the number of students who exercise 40-50 minutes each day.

(D) There are thrice as many students who exercise 40-50 minutes each day as the number of students who exercise 0-10 minutes each day.

18. A single letter is selected at random from the word 'Mathematics'. The probability that it is a vowel is :

(A) $\frac{11}{4}$ (B) $\frac{5}{11}$ (C) $\frac{4}{11}$ (D) $\frac{7}{11}$

Direction for questions 19 and 20 : In question numbers 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (R) is true but (A) is false.

19. **(A) :** LCM of two numbers 90 and 24 is 360.

(R) : Difference of 90 and 24 is 66.

20. **(A) :** Vertices of rectangle ABCD are A(0, 0), B(4, 0), C(4, 3) and D(0, 3). Length of its diagonal is 5 units.

(R) : Diagonal AC = Diagonal BD or $\sqrt{(0-4)^2+(0-3)^2} = \sqrt{(4-0)^2+(3-0)^2}$.

SECTION – B

Section-B consists of 5 questions of 2 marks each.

21. Given the linear equation $9x = 2y + 5$, write another linear equation in these two variables, such that the geometrical representation of the pair so formed is :

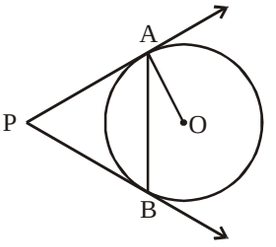
(*i*) intersecting lines, (*ii*) parallel lines.

22. Two circles touch each other internally at P. How many tangents can be drawn to the circles from an external point? What is the relation between the tangents ?

Or

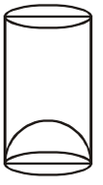
Two tangents PA and PB are drawn to a circle with centre O such that $\angle APB = 120°$. Prove that $OA = \sqrt{3}\, AP$.

23. In the given figure PA and PB are tangents to the circle with centre O. If $\angle APB = 60°$, find $\angle OAB$.



24. Find the value of : $\sin 30° \cdot \cos 60° - \tan 45° \cdot \sec 30° \cdot \operatorname{cosec} 60° + 2 \cdot \cot 45°$.

25. Find the actual capacity of a vessel shown in the figure, if the radius of base is 3.5 cm and height of the cylindrical part is 6 cm.



Or

A largest sphere is carved out of a cube of side 14 cm. Find the volume of the sphere.

SECTION – C

Section-C consists of 6 questions of 3 marks each.

26. Three alarm clocks ring at intervals of 4, 12 and 20 minutes respectively. If they start ringing together, after how much time will they next ring together ?

27. Find the zeroes of the following polynomial : $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$.

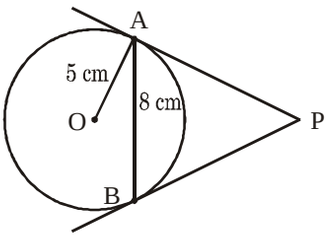
28. Check graphically, whether the pair of equations $x + 3y = 6$; $2x - 3y = 12$ is consistent. If so, then solve them graphically.

Or

Solve the following pair of linear equations by the substitution method :

$$\sqrt{3}x + \sqrt{5}y = 0 \; ; \; \sqrt{5}x - \sqrt{8}y = 0$$

29. In the given figure, AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP.



30. Prove that $(\text{cosec}\ \theta - \cot\ \theta)^2 = \dfrac{1 - \cos\ \theta}{1 + \cos\ \theta}$.

Or

Prove that : $\dfrac{\sin\ \theta}{\cot\ \theta + \text{cosec}\ \theta} = 2 + \dfrac{\sin\ \theta}{\cot\ \theta - \text{cosec}\ \theta}$.

31. A bag contains 1000 pens with different colours. 100 pens are taken out from the bag, out of which 20 were red, 35 were green, 15 were blue and remaining were black in colour. A pen is picked up from these 100 pens. What is the probability that the pen is
(*a*) not red (*b*) green ?

SECTION – D

Section-D consists of 4 questions of 5 marks each.

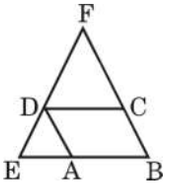
32. Find the value of K for which the quadratic equation $(2K + 1)x^2 - (7K + 2)x + (7K - 3) = 0$ has equal roots.

Or

Find the values of *k*, for which the given quadratic equation has equal roots :

$$4x^2 + kx + 6 = 0.$$

33. In a parallelogram ABCD, from vertex D, a line is drawn which intersects produced BA and BC at E and F respectively. Prove that $\dfrac{AD}{AE} = \dfrac{FB}{BE} = \dfrac{FC}{CD}$.



34. A medicine capsule is in the form of a cylinder with two hemispherical ends. The radius of the capsule is 2.8 mm and length of capsule is 14 mm. Find its total surface area. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Or

Water in a canal, 8 m wide and 6 m deep, is flowing with a speed of 12 km/hour. How much area will it irrigate in one hour, if 0.05 m of standing water is required ?

35. Determine the mean of the following distribution :

Marks	No. of students
Below 10	5
Below 20	9
Below 30	17
Below 40	29
Below 50	45
Below 60	60
Below 70	70
Below 80	78
Below 90	83
Below 100	85

SECTION – E

Case study based questions are compulsory.

36. **Pollution — A Major Problem**

One of the major serious problems that the world is facing today is the environmental pollution. Common types of pollution include light, noise, water and air pollution.

In a school, students thought of planting trees in and around the school to reduce noise pollution and air pollution.

Condition I : It was decided that the number of trees that each section of each class will plant be the same as the class in which they are studying, *e.g.*, a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on a section of class XII will plant 12 trees.

Condition II : It was decided that the number of trees that each section of each class will plant be the double of the class in which they are studying, *e.g.*, a section of class I will plant 2 trees, a section of class II will plant 4 trees and so on a section of class XII will plant 24 trees.

(*i*) Refer to Condition I — If there is only one section of each class, how many trees will be planted by the students ?

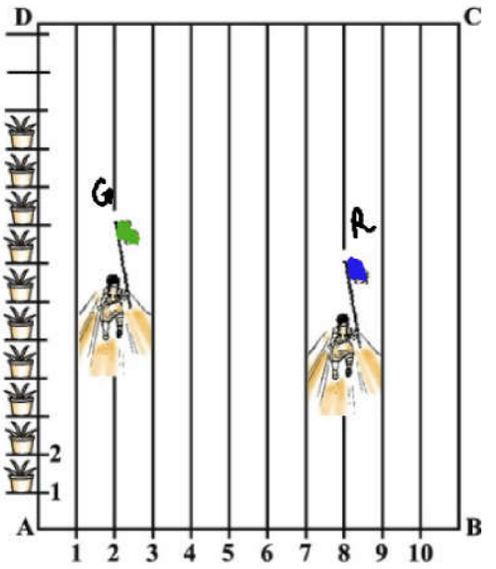
(*ii*) Refer to Condition II — If there is only one section of each class, how many trees will be planted by the students ?

(*iii*) Refer to Condition I — If there are two sections of each class, how many trees will be planted by the students?

Or

Refer to Condition II — If there are three sections of each class, how many trees will be planted by the students?

37. In order to conduct Sports Day activities in your school, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground ABCD, 100 flowerpots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs 1/4th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5th distance AD on the eighth line and posts a red flag.



(*i*) Find the position of green flag.

(*ii*) Find the position of red flag.

(*iii*) What is the distance between both the flags?

Or

If Joy has to post a flag at one-fourth distance from green flag ,in the line segment joining the green and red flags, then where should he post his flag?

38. **Statue of Unity**

It is a colossal statue of Indian statesman and independence activist Sardar Vallabh bhai Patel, who was the first Deputy Prime Minister and Home minister of independent India. Patel was highly respected for his leadership in uniting the 562 princely states of India to form the single Union of India. It is located in the state of Gujarat and it is the world's tallest statue. For a person standing at a point A, 240 m from the centre P of the base of the statue, the angle of elevation to the top B of the statue is 45°.

(*i*) Draw a well-labelled figure based on the above information.

(*ii*) How tall is the statue ?

(*iii*) A cop in helicopter, near the top of the statue, notices a car wreck some distance from the statue. If the angle of depression from the cop's eyes to the wreck is 60°, how far away is the accident from the centre of base of the statue?

Or

Find the distance of the car from the cop.