

## CBSE SAMPLE QUESTION PAPER (2023-24) with MARKING SCHEME <br> Mathematics (Basic)

Time Allowed : 3 Hours
CLASS-X
Maximum Marks : 80

## General Instructions:

1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section E has 3 sourced based/Case Based/passage based/integrated units of assessment (4 marks each) with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E .
8. Draw neat figures wherever required. Take $\pi=\frac{22}{7}$ wherever required if not stated.

## SECTION 'A'

1. If two positive integers $a$ and $b$ are written as $a=x^{3} y^{2}$ and $b=x y^{3} ; x, y$ are prime numbers, then $\operatorname{HCF}(a, b)$ is :
(a) $x y$
(b) $x y^{2}$
(c) $x^{3} y^{3}$
(d) $x^{2} y^{2}$

Ans. (b) $x y^{2}$
2. The LCM of smallest two-digit composite number and smallest composite number is:
(a) 12
(b) 4
(c) 20
(d) 44

Ans. (c) 20
3. If $x=3$ is one of the roots of the quadratic equation $x^{2}-2 k x-6=0$, then the value of $k$ is
(a) $-\frac{1}{2}$
(b) $\frac{1}{2}$
(c) 3
(d) 2

Ans. $\frac{1}{2}$
4. The pair of equations $\mathbf{y}=\mathbf{0}$ and $\mathbf{y}=-\mathbf{7}$ has :
(a) One solution
(b) Two solutions
(c) Infinitely many solutions
(d) No solution

Ans. (d) No solution
5. Value(s) of $k$ for which the quadratic equation $2 x^{2}-k x+k=0$ has equal roots is:
(a) 0 only
(b) 4
(c) 8 only
(d) 0,8

Ans. (d) 0, 8
6. The distance of the point $(3,5)$ from $x$-axis is $k$ units, then $k$ equals :
(a) 3
(b) -3
(c) 5
(d) -5

Ans. (c) 5
(a) $\triangle \mathrm{PQR} \sim \Delta \mathrm{CAB}$
(b) $\triangle \mathrm{PQR} \sim \triangle \mathrm{ABC}$
(c) $\triangle \mathrm{CBA} \sim \Delta \mathrm{PQR}$
(d) $\triangle \mathrm{BCA} \sim \triangle \mathrm{PQR}$

Ans. (a) $\Delta \mathrm{PQR} \sim \Delta \mathrm{CAB}$
8. Which of the following is NOT a similarity criterion ?
(a) AA
(b) SAS
(c) AAA
(d) RHS

Ans. (d) RHS
9. In figure, if TP and TQ are the two tangents to a circle with centre $O$ so that $\angle P O Q=110^{\circ}$, then $\angle P T Q$ is equal to

(a) $60^{\circ}$
(b) $70^{\circ}$
(c) $80^{\circ}$
(d) $90^{\circ}$

Ans. (b) $70^{\circ}$
10. If $\cos A=\frac{4}{5}$ then the value of $\tan A$ is :
(a) $\frac{3}{5}$
(b) $\frac{3}{4}$
(c) $\frac{4}{3}$
(d) $\frac{1}{8}$

Ans. (b) $\frac{3}{4}$
11. If the height of the tower is equal to the length of its shadow, then the angle of elevation of the sun is $\qquad$ .
(a) $30^{\circ}$
(b) $45^{\circ}$
(c) $60^{\circ}$
(d) $90^{\circ}$

Ans. (b) $45^{\circ}$
12. $1-\cos ^{2} A$ is equal to
(a) $\sin ^{2} A$
(b) $\tan ^{2} \mathrm{~A}$
(c) $1-\sin ^{2} \mathrm{~A}$
(d) $\sec ^{2} A$

Ans. (a) $\sin ^{2} A$ 1
13. The radius of a circle is same as the side of a square. Their perimeters are in the ratio
(a) $1: 1$
(b) $2: \pi$
(c) $\pi: 2$
(d) $\sqrt{\pi}: 2$

Ans. (c) $\pi: 2$
14. The area of the circle is 154 cm 2 . The radius of the circle is
(a) 7 cm
(b) 14 cm
(c) 3.5 cm
(d) 17.5 cm

Ans. (a) 7 cm
15. When a dice is thrown once, the probability of getting an even number less than 4 is
(a) $\frac{1}{4}$
(b) 0
(c) $\frac{1}{2}$
(d) $\frac{1}{6}$

Ans. (d) $\frac{1}{6}$
16. For the following distribution :

| Class | $0-5$ | $5-10$ | $10-15$ | $15-20$ | $20-25$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 10 | 15 | 12 | 20 | 9 |

The lower limit of modal class is:
(a) 15
(b) 25
(c) 30
(d) 35

Ans. (a) 15
17. A rectangular sheet of paper $40 \mathrm{~cm} \times 22 \mathrm{~cm}$, is rolled to form a hollow cylinder of height $\mathbf{4 0} \mathbf{~ c m}$. The radius of the cylinder (in $\mathbf{c m}$ ) is :
(a) 3.5
(b) 7
(c) 807
(d) 5

Ans. (a) 3.5
18. Consider the following frequency distribution :

| Class | $\mathbf{0 - 6}$ | $\mathbf{6 - 1 2}$ | $12-18$ | $18-24$ | $24-30$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 12 | 10 | 15 | 8 | 11 |

The median class is :
(a) 6-12
(b) 12-18
(c) 18-24
(d) $24-30$

Ans. (b) 12-18
19. Assertion (A) : The point $(0,4)$ lies on $y$-axis.

Reason ( $\mathbf{R}$ ) : The $x$ coordinate of the point on $y$-axis is zero.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertions (A) is true but reason (R) is false.
(d) Assertions (A) is false but reason (R) is true.

Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
20. Assertion (A) : The HCF of two numbers is 5 and their product is 150 . Then their LCM is 40 .
Reason (R) : For any two positive integers a and $b, \operatorname{HCF}(a, b) \times \operatorname{LCM}(a, b)=a \times b$.
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason $(R)$ is not the correct explanation of assertion (A).
(c) Assertions (A) is true but reason (R) is false.
(d) Assertions (A) is false but reason (R) is true.

Ans. (d) Assertions (A) is false but reason (R) is true.

## SECTION 'B'

21. Find whether the following pair of linear equations is consistent or inconsistent :

$$
\begin{aligned}
& 3 x+2 y=8 \\
& 6 x-4 y=9
\end{aligned}
$$

Ans. $3 x+2 y=8$
$6 x-4 y=9$
$\mathrm{a}_{1}=3, \mathrm{~b}_{1}=2, \mathrm{c}_{1}=8$
$a_{2}=6, b_{2}=4, c_{2}=9$
$\frac{a_{1}}{a_{2}}=\frac{3}{6}=\frac{1}{2}, \frac{b_{1}}{b_{2}}=\frac{2}{-4}=\frac{-1}{2}, \frac{c_{1}}{c_{2}}=\frac{8}{9}$
$\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}$
The given pair of linear equations are consistent.
22. In the given figure, if $A B C D$ is a trapezium in which $A B\|C D\| E F$, then prove that $\frac{\mathrm{AE}}{\mathrm{ED}}=\frac{\mathrm{BF}}{\mathrm{FC}}$.


Ans. Given : $A B\|C D\| E F$
To prove: $\frac{\mathrm{AE}}{\mathrm{ED}}=\frac{\mathrm{BF}}{\mathrm{FC}}$
Construction : J oin BD to intersect EF at G.
Proof: In $\triangle A B D$,

$$
E G \| A B \quad(E F \| A B)
$$



$$
\frac{\mathrm{AE}}{\mathrm{ED}}=\frac{\mathrm{BG}}{\mathrm{GD}} \quad \text { (by BPT) }
$$

In $\triangle \mathrm{DBC}$,

$$
\begin{array}{ll}
G F \| C D & (E F \| C D) \\
\frac{B F}{F C}=\frac{B G}{G D} & (\text { by } B P T)
\end{array}
$$$1 / 2$

From (1) and (2)

$$
\frac{\mathrm{AE}}{\mathrm{ED}}=\frac{\mathrm{BF}}{\mathrm{FC}}
$$

$$
1 / 2
$$

## Or

In figure, if $A D=6 \mathrm{~cm}, \mathrm{DB}=9 \mathrm{~cm}, \mathrm{AE}=8 \mathrm{~cm}$ and $E C=12 \mathrm{~cm}$ and $\angle A D E=48^{\circ}$. Find $\angle A B C$.


Ans. Given : $\mathrm{AD}=6 \mathrm{~cm}, \mathrm{DB}=9 \mathrm{~cm}$

$$
\mathrm{AE}=8 \mathrm{~cm}, \mathrm{EC}=12 \mathrm{~cm}, \angle \mathrm{ADE}=48^{\circ}
$$

To find : $\angle A B C=$ ?
Proof: In $\triangle \mathrm{ABC}, \quad \frac{\mathrm{AD}}{\mathrm{DB}}=\frac{6}{9}=\frac{2}{3}$

$$
\begin{equation*}
\frac{\mathrm{AE}}{\mathrm{EC}}=\frac{8}{12}=\frac{2}{3} \tag{1}
\end{equation*}
$$



From (1) and (2),

$$
\begin{array}{rlrl} 
& \frac{\mathrm{AD}}{\mathrm{DB}} & =\frac{\mathrm{AE}}{\mathrm{EC}} & \\
\mathrm{DE} \| \mathrm{BC} & & \text { (Converse of BPT) } \\
\angle \mathrm{ADE} & =\angle \mathrm{ABC} & & \text { (Corresponding angles) } \\
\Rightarrow \quad \angle \mathrm{ABC} & =48 . & &
\end{array}
$$

23. The length of a tangent from a point $A$ at distance 5 cm from the centre of the circle is $\mathbf{4} \mathbf{~ c m}$. Find the radius of the circle.
Ans. In $\triangle \mathrm{OTA}, \angle \mathrm{OTA}=90$.
By Pythagoras theorem

$$
\begin{aligned}
\mathrm{OA}^{2} & =\mathrm{OT}^{2}+\mathrm{AT}^{2} \\
(5)^{2} & =\mathrm{OT}^{2}+(4)^{2} \\
25-16 & =\mathrm{OT}^{2} \\
9 & =\mathrm{OT}^{2} \\
\mathrm{OT} & =3 \mathrm{~cm} \\
\text { Radius of circle } & =3 \mathrm{~cm} .
\end{aligned}
$$


24. Evaluate : $\sin ^{\mathbf{2}} \mathbf{6 0 ^ { \circ }}+\mathbf{2} \tan 45^{\circ}-\cos ^{2} \mathbf{3 0 ^ { \circ }}$.

Ans. $\sin ^{2} 60^{\circ}+2 \tan 45^{\circ}-\cos ^{2} 30^{\circ}$

$$
\begin{align*}
& =\left(\frac{\sqrt{3}}{2}\right)^{2}+2(1)-\left(\frac{\sqrt{3}}{2}\right)^{2}  \tag{1}\\
& =\frac{3}{4}+2-\frac{3}{4} \\
& =2 \tag{1}
\end{align*}
$$

25. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm ?
Ans. Area of the circle $=$ Sum of areas of 2 circles

$$
\begin{array}{rlrl}
\pi \mathrm{R}^{2} & =\pi(40)^{2}+\pi(9)^{2} & 1 / 2 \\
\pi \mathrm{R}^{2} & =\pi \times\left(40^{2}+9\right)^{2} & 1 / 2 \\
\mathrm{R}^{2} & =1600+81 & \\
\mathrm{R}^{2} & =1681 & \\
\mathrm{R} & =41 \mathrm{~cm} . & 1 / 2 \\
\text { Diameter of given circle } & =41 \times 2=82 \mathrm{~cm} & \mathbf{O r} &
\end{array}
$$

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. (Use $\pi=3.14$ )
Ans. Radius of circle $=10 \mathrm{~cm}, \theta=90^{\circ}$
Area of minor segment $=\frac{\theta}{360^{\circ}} \pi r^{2}$ - Area of $\Delta$

$$
\begin{array}{lr}
=\frac{\theta}{360^{\circ}} \times \pi r^{2}-\frac{1}{2} \times b \times h & 1 / 2 \\
=\frac{90^{\circ}}{360^{\circ}} \times 3.14 \times 10 \times 10-\frac{1}{2} \times 10 \times 10 & 1 / 2 \\
=\frac{314}{4}-50 & \\
=78.5-50=28.5 \mathrm{~cm}^{2} & 1 / 2
\end{array}
$$

Area of minor segment $=28.5 \mathrm{~cm}^{2} \quad 1 / 2$

## SECTION 'C'

26. Prove that $\sqrt{3}$ is an irrational number.

Ans. Let us assume that $\sqrt{3}$ be a rational number.

$$
\begin{equation*}
\sqrt{3}=\frac{a}{b} \text { where } \mathrm{a} \text { and } \mathrm{b} \text { are co-prime. } \tag{1}
\end{equation*}
$$

Squaring both sides,

$$
\begin{align*}
(\sqrt{3})^{2} & =\left(\frac{a}{b}\right)^{2} \\
3 & =\frac{a^{2}}{b^{2}} \Rightarrow a^{2}=3 b^{2}
\end{align*}
$$

$a^{2}$ is divisible by 3 so a is also divisible by 3.
Let $a=3 c$ for any integer $c$.

$$
\begin{aligned}
(3 c)^{2} & =3 b^{2} \\
9 c^{2} & =3 b^{2}
\end{aligned}
$$

$$
\begin{equation*}
b^{2}=3 c^{2} \tag{2}
\end{equation*}
$$

Since $a^{2}$ is divisible by 3 so, $b$ is also divisible by 3 .
From (1) and (2), we can say that 3 in a factor of $a$ and $b$
which is contradicting the fact that a and b are co- prime.
Thus, our assumption that $\sqrt{3}$ is a rational number is wrong.
Hence, $\sqrt{3}$ is an irrational number.
27. Find thezeroes of the quadratic polynomial $4 s^{\mathbf{2}}-\mathbf{4 s}+1$ and verify the relationship between the zeroes and the coefficients.

$$
\begin{align*}
\text { Ans. } \begin{aligned}
& P(S)=4 S^{2}-4 S+1 \\
& 4 S^{2}-2 S-2 S+1=0 \\
& 2 S(2 S-1)-1(2 S-1)=0 \\
&(2 S-1)(2 S-1)=0 \\
& S=\frac{1}{2} \quad S=\frac{1}{2} \\
& a=4, b=-4, c=1, \alpha=\frac{1}{2}, \beta=\frac{1}{2} \\
& \alpha+\beta=\frac{-b}{a}
\end{aligned}
\end{align*}
$$

LHS $=\alpha+\beta=\frac{1}{2}+\frac{1}{2}=1$, RHS $=\frac{-b}{a}=\frac{(-4)}{4}=1$, hence proved.

$$
\begin{equation*}
\alpha \beta=\frac{c}{a} \tag{1}
\end{equation*}
$$

LHS $=\alpha \beta=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$, RHS $=\frac{c}{a}=\frac{1}{4}$, hence proved.
28. The coach of a cricket team buys 4 bats and 1 ball for ₹ 2050. Later, she buys 3 bats and 2 balls for ₹ $\mathbf{1 6 0 0}$. Find the cost of each bat and each ball.
Ans. Let cost of one bat be ₹ x .
Let cost of one ball be $₹ y$.
According to the question,

$$
\begin{align*}
& 4 x+1 y=2050  \tag{1}\\
& 3 x+2 y=1600 \tag{2}
\end{align*}
$$

From (1),

$$
\begin{align*}
4 x+1 y & =2050 \\
y & =2050-4 x
\end{align*}
$$

Substitute value of $y$ in (2),

$$
\begin{align*}
3 x+2(2050-4 x) & =1600 \\
3 x+4100-8 x & =1600 \\
-5 x & =-2500 \\
x & =500
\end{align*}
$$

Substitute value of $x$ in (1),

$$
\begin{align*}
4 x+1 y & =2050 \\
4(500)+y & =2050 \\
2000+y & =2050 \\
y & =50
\end{align*}
$$

$\begin{array}{lll}\text { Hence, } & \text { cost of one bat }=₹ 500 & 1 / 2\end{array}$
Cost of one ball $=₹ 50$

## Or

A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ $\mathbf{2 1}$ for the book she kept for five days. Find the fixed charge and the charge for each extra day.
Ans. Let the fixed charge for first 3 days $=₹ x$
and additional charge after 3 days $=₹$ y
According to the question,

$$
\begin{align*}
& x+4 y=27  \tag{1}\\
& x+2 y=21 \tag{2}
\end{align*}
$$

Subtract eqn. (2) from (1),

$$
\begin{align*}
2 y & =6 \\
y & =3 \tag{1}
\end{align*}
$$

Substitute value of $y$ in (2),

$$
\begin{align*}
x+2(3) & =21 \\
x & =21-6 \\
x & =15 \tag{1}
\end{align*}
$$

Fixed charge $=₹ 15$
Additional charge per day $=₹ 3$
29. A circle touches all the four sides of quadrilateral ABCD. Prove that $A B+C D=A D+B C$.
Ans. Given circle touching sides of $A B C D$ at $P, Q, R$ and $S$.
To prove: $A B+C D=A D+B C$
Proof:

$$
\begin{align*}
& \mathrm{AP}=\mathrm{AS} \ldots(1) \text { tangents from an external point } \\
& \mathrm{PB}=\mathrm{BQ} \ldots(2) \text { to a circle are equal in length } \\
& \mathrm{DR}=\mathrm{DS} \ldots(3) \\
& \mathrm{CR}=\mathrm{CQ} \ldots(4)
\end{align*}
$$

Adding eqn. (1), (2), (3) and (4),


$$
\begin{align*}
A P+B P+D R+C R & =A S+D S+B Q+C Q \\
A B+D C & =A D+B C \tag{1}
\end{align*}
$$

30. Prove that $(\operatorname{cosec} \theta-\cot \theta)^{\mathbf{2}}=\frac{\mathbf{1 - \operatorname { c o s }} \theta}{1+\cos \theta}$

Ans. $\quad(\operatorname{cosec} \theta-\cot \theta)^{2}=\frac{1-\cos \theta}{1+\cos \theta}$

$$
\begin{array}{rlr}
\text { LHS } & =(\operatorname{cosec} \theta-\cot \theta)^{2} & 1 / 2 \\
& =\left(\frac{1}{\sin \theta}-\frac{\cos \theta}{\sin \theta}\right)^{2} & 1 / 2 \\
& =\left(\frac{1-\cos \theta}{\sin \theta}\right)^{2} & \\
& =\frac{(1-\cos \theta)^{2}}{\sin ^{2} \theta} & 1
\end{array}
$$

$$
\begin{align*}
& =\frac{(1-\cos \theta)^{2}}{(1-\cos \theta)(1+\cos \theta)} \\
& =\frac{1-\cos \theta}{1-\cos \theta}=\text { RHS }  \tag{1}\\
\text { LHS } & =\text { RHS. Hence proved. }
\end{align*}
$$

Or
Prove that $\sec \mathbf{A}(\mathbf{1}-\sin \mathbf{A})(\sec \mathbf{A}+\tan \mathbf{A})=\mathbf{1}$.
Ans. $\sec A(1-\sin A)(\sec A+\tan A)=1$

$$
\begin{aligned}
\text { LHS } & =\frac{1}{\cos \mathrm{~A}}(1-\sin \mathrm{A})\left(\frac{1}{\cos \mathrm{~A}}+\frac{\sin \mathrm{A}}{\cos \mathrm{~A}}\right) \\
& =\frac{(1-\sin \mathrm{A})}{\cos \mathrm{A}} \frac{(1+\sin \mathrm{A})}{\cos \mathrm{A}} \\
& =\frac{(1-\sin \mathrm{A})(1+\sin \mathrm{A})}{\cos ^{2} \mathrm{~A}} \\
& =\frac{1-\sin ^{2} \mathrm{~A}}{\cos ^{2} \mathrm{~A}} \\
& =\frac{\cos ^{2} \mathrm{~A}}{\cos ^{2} \mathrm{~A}} \\
& =1=\text { RHS } \\
\text { LHS } & =\text { RHS. Hence proved. }
\end{aligned}
$$

31. A bag contains 6 red, 4 black and some white balls.
(i) Find the number of white balls in the bag if the probability of drawing a white ball is $\frac{1}{3}$.
(ii) How many red balls should be removed from the bag for the probability of drawing a white ball to be $\frac{1}{2}$ ?
Ans. (i) Red balls $=6$, Black balls $=4$, White balls $=x$

$$
\begin{array}{rlrl} 
& \mathrm{P}(\text { white ball }) & =\frac{x}{10+x}=\frac{1}{3} \\
\Rightarrow \quad 3 \mathrm{x} & =10+\mathrm{x} \Rightarrow \mathrm{x}=5 \text { white balls }
\end{array}
$$

(ii) Let y red balls be removed, black balls $=4$, white balls $=5$

$$
\begin{align*}
& \mathrm{P}(\text { white balls }) & =\frac{5}{(6-y)+4+5}=\frac{1}{2}  \tag{1}\\
\Rightarrow \quad & \frac{5}{15-y} & =\frac{1}{2} \Rightarrow 10=15-\mathrm{y} \Rightarrow \mathrm{y}=5
\end{align*}
$$

So, 5 balls should be removed.

## SECTION 'D'

32. A train travels 360 km at a uniform speed. If the speed had been $5 \mathrm{~km} / \mathrm{h}$ more, it would have taken 1 hour less for the same journey. Find the speed of the train. Ans. Let the speed of train be $x \mathrm{~km} / \mathrm{hr}$.

$$
\begin{aligned}
\text { Distance } & =360 \mathrm{~km} \\
\text { Speed } & =\frac{\text { Distance }}{\text { Time }}
\end{aligned}
$$

$$
\begin{array}{rlrl}
\text { Time } & =\frac{360}{x} & 1 / 2 \\
\text { New speed } & =(x+5) \mathrm{km} / \mathrm{hr} & & \\
\text { Time } & =\frac{\mathrm{D}}{5} & & \\
x+5 & =\frac{360}{\left(\frac{360}{x}-1\right)} & & \\
x+5\left(\frac{360}{x}-1\right) & =360 & & \\
(x+5)(360-x) & =360 x & & \\
-x^{2}-5 x+1800 & =0 & \\
x^{2}-5 x-1800 & =0 & \\
x^{2}+45 x-40 x-1800 & =0 & \\
x(x+45)-40(x+45) & =0 & & \\
(x+45)(x-40) & =0 & x-40=0 & \\
x+45 & =0 & & \\
x & =-45 & & 1
\end{array}
$$

## Or

A motor boat whose speed is $18 \mathrm{~km} / \mathrm{h}$ in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
Ans. Let the speed of the stream $=x \mathrm{~km} / \mathrm{hr} \quad 1 / 2$
Speed of boat $=18 \mathrm{~km} / \mathrm{hr}$
Upstream speed $=(18-x) \mathrm{km} / \mathrm{hr}$ Downstream speed $=(18+x) \mathrm{km} / \mathrm{hr}$ 1/2

$$
\text { Time taken }(\text { upstream })=\frac{24}{(18-x)}
$$

Time taken $($ downstream $)=\frac{24}{(18+x)}$
According to the question,

$$
\begin{align*}
\frac{24}{(18-x)} & =\frac{24}{(18+x)}+1  \tag{1}\\
\frac{24}{(18-x)}-\frac{24}{(18+x)} & =1 \\
24(18+x)-24(18-x) & =(18-x)(18+x) \\
24(2 x) & =324-x^{2} \\
48 x-324+x^{2} & =0 \\
x^{2}+48 x-324 & =0 \\
x^{2}-6 x+54 x-324 & =0 \\
x(x-6)+54(x-6) & =0 \\
(x-6)(x+54) & =0
\end{align*}
$$

$$
\begin{array}{rlrl}
x-6 & =0 & x+54 & =0 \\
x & =6 & x & =-54
\end{array}
$$

Speed cannot be negative Speed of stream $=6 \mathrm{~km} / \mathrm{hr}$
33. Prove that If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
In $\triangle P Q R, S$ and $T$ are points on $P Q$ and $P R$ respectively. $\frac{P S}{S Q}=\frac{P T}{T R}$ and $\triangle P S T=$ $\triangle P R Q$. Prove that $P Q R$ is an isosceles triangle.
Ans. Given : $\triangle A B C, D E \| B C$
To prove : $\frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$
Construction : Join BE and CD.
Draw $D M \perp A C$ and $E N \perp A B$.
Proof : Area of $\triangle A D E=\frac{1}{2} \times b \times h$

$$
\begin{align*}
& =\frac{1}{2} \times \mathrm{AD} \times \mathrm{EN}  \tag{1}\\
\text { Area of }(\triangle \mathrm{DBE}) & =\frac{1}{2} \times \mathrm{DB} \times \mathrm{EN} \tag{2}
\end{align*}
$$



Divide eqn. (1) by (2),

$$
\begin{align*}
\frac{\operatorname{ar} \triangle \mathrm{ADE}}{\operatorname{ar} \triangle \mathrm{DBE}} & =\frac{\frac{1}{2} \times \mathrm{AD} \times \mathrm{EN}}{\frac{1}{2} \times \mathrm{DB} \times \mathrm{EN}}=\frac{\mathrm{AD}}{\mathrm{DB}}  \tag{3}\\
\text { Area of } \triangle \mathrm{ADE} & =\frac{1}{2} \times \mathrm{AE} \times \mathrm{DM}  \tag{4}\\
\text { Area of } \triangle \mathrm{DEC} & =\frac{1}{2} \times \mathrm{EC} \times \mathrm{DM} \tag{5}
\end{align*}
$$

Divide eqn. (4) by (5),

$$
\frac{\operatorname{ar} \triangle \mathrm{ADE}}{\operatorname{ar} \triangle \mathrm{DEC}}=\frac{\frac{1}{2} \times \mathrm{AE} \times \mathrm{DM}}{\frac{1}{2} \times \mathrm{EC} \times \mathrm{DM}}=\frac{\mathrm{AE}}{\mathrm{EC}}
$$

$\triangle \mathrm{BDE}$ and $\triangle \mathrm{DEC}$ are on the same base DE and between same parallel lines EC and DE.


Hence, $\triangle \mathrm{PQR}$ is isosceles.
34. A medicine capsule is in the shape of a cylinder with two hemispheres stuck at each of its ends. The length of the entire capsule is $\mathbf{1 4} \mathbf{~ m m}$ and the diameter of the capsule is $\mathbf{5} \mathbf{~ m m}$. Find its surface area.


Ans. Diameter of cylinder and hemisphere $=5 \mathrm{~mm}$ radius, $(r)=\frac{5}{2}$
Total length $=14 \mathrm{~mm}$
Height of cylinder $=14-5=9 \mathrm{~mm}$
CSA of cylinder $=2 . \mathrm{rh}$

$$
\begin{align*}
= & 2 \times \frac{22}{7} \times \frac{5}{2} \times 9 \\
& =\frac{990}{7} \mathrm{~mm}^{2} \tag{1}
\end{align*}
$$

CSA of hemispheres $=2 \times r^{2}$

$$
\begin{aligned}
& =2 \times \frac{22}{7} \times\left(\frac{5}{2}\right)^{2} \\
& =\frac{275}{7} \mathrm{~mm}^{2}
\end{aligned}
$$

CSA of 2 hemispheres $=2 \times \frac{275}{7}$

$$
\begin{equation*}
=\frac{550}{7} \mathrm{~mm}^{2} \tag{1}
\end{equation*}
$$

Total area of capsule $=\frac{990}{7}+\frac{550}{7}$

$$
=\frac{1540}{7}
$$

$$
=220 \mathrm{~mm}^{2}
$$

Or
A gulab jamun, contains sugar syrup up to about 30\% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like cylinder with two hemispherical ends with length $5 \mathbf{c m}$ and diameter $\mathbf{2 . 8} \mathbf{c m}$.


Ans. $\quad$ Diameter of cylinder $=2.8 \mathrm{~cm}$
Radius of cylinder $=\frac{2.8}{2}=1.4 \mathrm{~cm}$
Radius of cylinder $=$ Radius of hemisphere $=1.4 \mathrm{~cm}$

$$
\begin{aligned}
\text { Height of cylinder } & =5-2.8 \\
& =2.2 \mathrm{~cm}
\end{aligned}
$$

Volume of 1 gulab jamun $=$ Volume of cylinder $+2 \times$ Volume of hemisphere

$$
\begin{align*}
& =\pi r^{2} h+2 \times \frac{2}{3} \pi r^{3}  \tag{1}\\
& =\frac{22}{7} \times(1.4)^{2} \times 2.2+2 \times \frac{2}{3} \times \frac{22}{7} \times(1.4)^{3} \\
& =13.55+11.50 \\
& =25.05 \mathrm{~cm}^{3} \\
& \mathrm{~s}=45 \times 25.05  \tag{1}\\
& \mathrm{~s}=30 \% \times 45 \times 25.05  \tag{1}\\
& =\frac{30}{100} \times 45 \times 25.05 \\
& =338.175 \mathrm{~cm}^{3} \\
& =338 \mathrm{~cm}^{3}
\end{align*}
$$

Volume of 45 gulab jamuns $=45 \times 25.05$ Syrup in 45 gulab jamuns $=30 \% \times 45 \times 25.05$
Su
35. The following table gives the distribution of the life time of $\mathbf{4 0 0}$ neon lamps :

| Life time (in hours) | Number of lamps |
| :---: | :---: |
| $\mathbf{1 5 0 0}-2000$ | $\mathbf{1 4}$ |
| $2000-2500$ | 56 |
| $2500-3000$ | 60 |
| $\mathbf{3 0 0 0}-\mathbf{3 5 0 0}$ | $\mathbf{8 6}$ |
| $\mathbf{3 5 0 0}-\mathbf{4 0 0 0}$ | $\mathbf{7 4}$ |
| $\mathbf{4 0 0 0}-\mathbf{4 5 0 0}$ | $\mathbf{6 2}$ |
| $\mathbf{4 5 0 0}-\mathbf{5 0 0 0}$ | $\mathbf{4 8}$ |

Find the average life time of a lamp.
Ans.

| Life time <br> (in hours) | Number of lamps <br> $(f)$ | $\operatorname{Mid} x$ | $d$ | $f d$ |
| :---: | :---: | :---: | :---: | :---: |
| $1500-2000$ | 14 | 1750 | -1500 | -21000 |
| $2000-2500$ | 56 | 2250 | -1000 | -56000 |
| $2500-3000$ | 60 | 2750 | -500 | -3000 |
| $3000-3500$ | 86 | 3250 | 0 | 0 |
| $3500-4000$ | 74 | 3750 | 500 | 37000 |
| $4000-4500$ | 62 | 4250 | 1000 | 62000 |
| $4500-5000$ | 48 | 4750 | 1500 | 72000 |
|  | 400 |  |  | 64000 |

$$
\begin{aligned}
\text { Mean } & =\mathrm{a}+\frac{\Sigma f d}{\Sigma f} \\
\mathrm{a} & =3250 \\
\text { Mean } & =3250+\frac{64000}{400} \\
& =3250+160=3410
\end{aligned}
$$

Average life of Iamp is 3410 hr .

## SECTION 'E’

36. CASE STUDY 1

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) In which year, the production is ₹ 29,200.
(ii) Find the production during 8th year.

Or
Find the production during first 3 years.
(iii) Find the difference of the production during 7th year and 4th year.

Ans.

$$
\begin{align*}
a_{6} & =16000 \\
a_{9} & =22600 \\
a+5 d & =16000  \tag{1}\\
a+8 d & =22600 \tag{2}
\end{align*}
$$

Substitute a = 1600-5d from (1),

$$
\begin{aligned}
16000-5 d+8 d & =22600 \\
3 d & =22600-16000 \\
3 d & =6600 \\
d & =\frac{6600}{3}=2200 \\
a & =16000-5(2200) \\
a & =16000-11000 \\
a & =5000 \\
a_{n} & =29200, a=5000, d=2200 \\
a_{n} & =a+(n-1) d \\
29200 & =5000+(n-1) 2200 \\
29200-5000 & =2200 \mathrm{n}-2200 \\
24200+2200 & =2200 \mathrm{n} \\
26400 & =2200 \mathrm{n} \\
\mathrm{n} & =\frac{264}{22} \\
\mathrm{n} & =12
\end{aligned}
$$

In 12th year the production was ₹ 29,200.

$$
\begin{aligned}
\mathrm{n} & =8, a=5000, \mathrm{~d}=2200 \\
\mathrm{a}_{\mathrm{n}} & =\mathrm{a}+(\mathrm{n}-1) \mathrm{d}
\end{aligned}
$$

$$
=5000+(8-1) 2200 \quad 1 / 2
$$

$$
=5000+7 \times 2200
$$

$$
=5000+15400
$$

$$
1 / 2
$$

$$
=20400
$$

The production during 8th year is 20,400.
Or

$$
\begin{align*}
\mathrm{n} & =8, \mathrm{a}=5000, \mathrm{~d}=2200 \\
\mathrm{~S}_{\mathrm{n}} & =\frac{n}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}] \\
& =\frac{3}{2}[2(5000)+(3-1) 2200] \\
\mathrm{S}_{3} & =\frac{3}{2}(10000+2 \times 2200)
\end{align*}
$$

$$
=\frac{3}{2}(10000+4400)
$$

$$
1 / 2
$$

$$
=3 \times 7200
$$

$$
=21600
$$1/2

The production during first 3 year is 21,600.
(ii)

$$
\begin{align*}
a_{4} & =a+3 d \\
& =5000+3(2200) \\
& =5000+6600 \\
& =11600 \\
a_{7} & =a+6 d \\
& =5000+6 \times 2200 \\
& =5000+13200 \\
& =18200 \\
a_{7}-a_{4} & =18200-11600=6600
\end{align*}
$$

37. CASE STUDY 2

Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house. Suppose the school is situated at the point $0, i . e .$, the origin, Alia's house is at A. Shagun's house is at B and library is at C. Based on the above information, answer the following questions.

(i) How far is Alia's house from Shagun's house?
(ii) How far is the library from Shagun's house?
(iii) Show that for Shagun, school is farther compared to Alia's house and library. Or
Show that Alia's house, Shagun's house and library for an isosceles right triangle. Ans. Coordinates of A $(2,3)$ Alia's house
Coordinates of $B(2,1)$ Shagun's house
Coordinates of C $(4,1)$ Library

$$
\begin{array}{rlr}
\mathrm{AB} & =\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} & \\
& =\sqrt{(2-2)^{2}+(1-3)^{2}} & 1 / 2 \\
& =\sqrt{(0)^{2}+(-2)^{2}} & \\
\mathrm{AB} & =\sqrt{0+4}=\sqrt{4}=2 \text { units } & 1 / 2
\end{array}
$$

Alia's house from Shagun's house is 2 units.
(ii) $\mathrm{C}(4,1), \mathrm{B}(2,1)$

$$
\begin{array}{rlr}
\mathrm{CB} & =\sqrt{(2-4)^{2}-(1-1)^{2}} & 1 / 2 \\
& =\sqrt{(-2)^{2}+(0)^{2}} & \\
& =\sqrt{4+0}=\sqrt{4}=2 \text { units } & 1 / 2
\end{array}
$$

(iii) $\mathrm{O}(0,0), \mathrm{B}(2,1)$

$$
\begin{align*}
\mathrm{OB} & =\sqrt{(2-0)^{2}+(1-0)^{2}} \\
& =\sqrt{2^{2}+1^{2}}=\sqrt{4+1}=5 \mathrm{units} \tag{1}
\end{align*}
$$

Distance between Alia's house and Shagun's house, $A B=2$ units
Distance between Library and Shagun's house, CB $=2$ units1/2
$O B$ is greater than $A B$ and $C B$, ..... 1/2

For Shagun, school $[O]$ is farther than Alia's house [A] and Library [C]

## Or

$C(4,1), A(2,3)$

$$
\begin{align*}
\mathrm{CA} & =\sqrt{(2-4)^{2}+(3-1)^{2}} \\
& =\sqrt{(-2)^{2}+2^{2}}=\sqrt{4+4}=\sqrt{8} \\
& =2 \sqrt{2} \text { units } \quad \mathrm{AC}^{2}=8 \tag{1}
\end{align*}
$$

Distance between Alia's house and Shagun's house, $\mathrm{AB}=2$ units
Distance between Library and Shagun's house, $\mathrm{CB}=2$ units

$$
\begin{align*}
\mathrm{AB}^{2}+\mathrm{BC}^{2} & =2^{2}+2^{2} \\
& =4+4=8=\mathrm{AC}^{2}
\end{align*}
$$

Therefore $A, B$ and $C$ form an isosceles right triangle.
38. CASE STUDY 3

A boy is standing on the top of light house. He observed that boat $P$ and boat $Q$ are approaching the light house from opposite directions. He finds that angle of depression of boat $P$ is $45^{\circ}$ and angle of depression of boat $Q$ is $30^{\circ}$. He also knows that height of the light house is $\mathbf{1 0 0} \mathbf{~ m}$.


Based on the above information, answer the following questions.
(i) What is the measure of $\angle A P D$ ?
(ii) If $\angle \mathrm{YAQ}=30^{\circ}$, then $\angle \mathrm{AQD}$ is also $30^{\circ}$. Why?
(iii) How far is boat $\mathbf{P}$ from the light house?

Or
How far is the boat $Q$ from the light house?
Ans. (i) $X Y \| P Q$ and $A P$ is transversal.

$$
\angle \mathrm{APD}=\angle \mathrm{PAX} \quad \text { (alternative interior angles) } \quad 1 / 2
$$

$$
\angle \mathrm{APD}=30^{\circ} \quad 1 / 2
$$

$$
\angle Y A Q=30^{\circ}
$$

$$
\angle \mathrm{AQD}=30^{\circ}
$$

Because $X Y \| P Q$ and $A Q$ is a transversal. So alternate interior angles are equal.

$$
\angle \mathrm{YAQ}=\angle \mathrm{AQD}
$$

(iii) In $\triangle A D P$

$$
\begin{aligned}
\tan 45^{\circ} & =\frac{100}{\mathrm{PD}} \\
1 & =\frac{100}{\mathrm{PD}} \\
\mathrm{PD} & =100 \mathrm{~m}
\end{aligned}
$$



Boat $P$ is 100 m from the light house.
Or
In $\triangle \mathrm{ADQ}$

$$
\begin{align*}
\tan 30^{\circ} & =\frac{100}{\mathrm{DQ}} \\
\frac{1}{\sqrt{3}} & =\frac{100}{\mathrm{DQ}} \\
\mathrm{DQ} & =100 \sqrt{3} \mathrm{~m}
\end{align*}
$$

Boat Q is $100 \sqrt{3} \mathrm{~m}$ from the light house.

