



**CBSE Sample
Question Paper
(2023-24)**

Mathematics

(Standard)

Class-X

Solved

(with Marking Scheme)

CBSE SAMPLE QUESTION PAPER (2023-24)
with MARKING SCHEME
Mathematics (Standard)

Time Allowed : 3 Hours

CLASS-X

Maximum Marks : 80

General Instructions :

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All questions are compulsory. However, an internal choice in 2 questions of 5 marks, 2 questions of 3 marks and 2 questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION 'A'

Section A consists of 20 questions of 1 mark each.

1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then the result obtained by dividing the product of the positive integers by the LCM (a, b) is

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

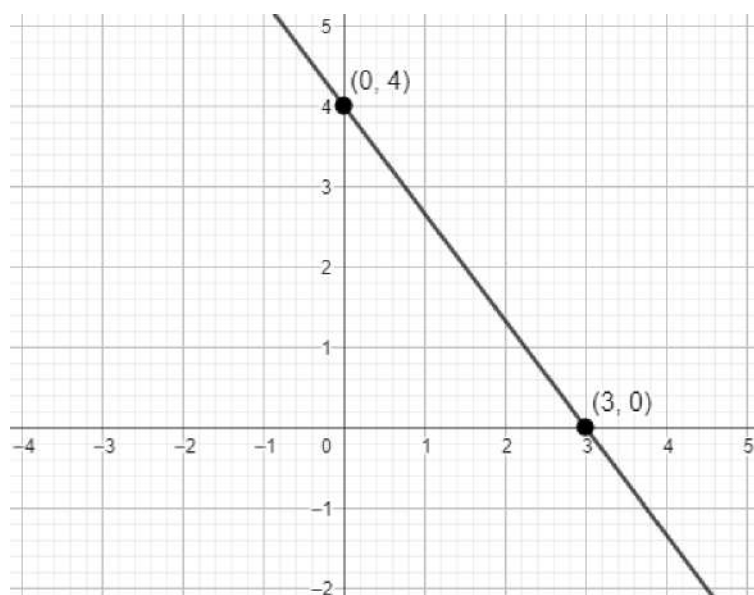
Ans. (b) xy^2

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2. The given linear polynomial $y = f(x)$ has

(a) 2 zeros (b) 1 zero and the zero is '3'

(c) 1 zero and the zero is '4' (d) No zero

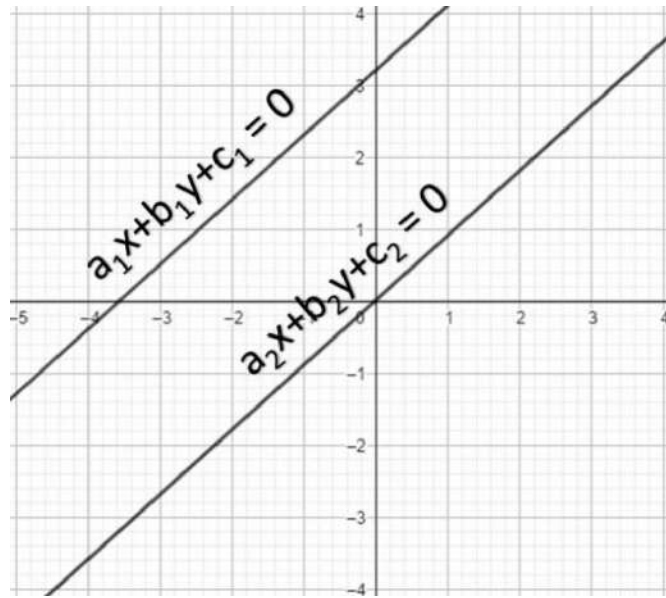


Ans. (b) 1 zero and the zero is '3'

1

3. The given pair of linear equations is non-intersecting. Which of the following statements is true?

- (a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (c) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (d) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$



Ans. (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ 1

4. Write the nature of roots of the quadratic equation $9x^2 - 6x - 2 = 0$.

- (a) No real roots (b) 2 equal real roots
(c) 2 distinct real roots (d) More than 2 real roots

Ans. (c) 2 distinct real roots 1

5. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is

- (a) 1 (b) -7 (c) 7 (d) 9

Ans. (c) 7 1

6. Find the ratio in which the line segment joining (2, -3) and (5, 6) is divided by x-axis.

- (a) 1 : 2 (b) 2 : 1 (c) 2 : 5 (d) 5 : 2

Ans. (a) 1 : 2 1

7. (x, y) is 5 unit from the origin. How many such points lie in the third quadrant?

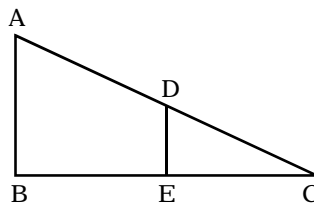
- (a) 0 (b) 1 (c) 2 (d) infinitely many

Ans. (d) infinitely many 1

8. In $\triangle ABC$, $DE \parallel AB$. If $AB = a$, $DE = x$, $BE = b$ and $EC = c$

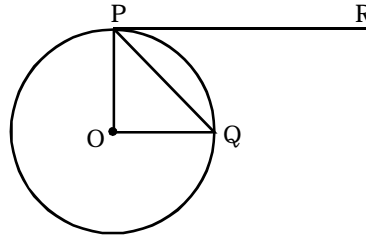
Express x in terms of a , b and c

- (a) $\frac{ac}{b}$ (b) $\frac{ac}{b+c}$ (c) $\frac{ab}{c}$ (d) $\frac{ab}{b+c}$



Ans. (b) $\frac{ac}{b+c}$ 1

9. If O is centre of a circle and Chord PQ makes an angle 50° with the tangent PR at the point of contact P, find the angle made by the chord at the centre.
 (a) 130° (b) 100° (c) 50° (d) 30°



Ans. (b) 100° 1

10. A Quadrilateral PQRS is drawn to circumscribe a circle. If $PQ = 12$ cm, $QR = 15$ cm and $RS = 14$ cm, find the length of SP.
 (a) 15 cm (b) 14 cm (c) 12 cm (d) 11 cm
 Ans. (d) 11 cm 1

11. Given that $\sin \theta = \frac{a}{b}$, find $\cos \theta$.

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{b}{a}$ (c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

Ans. (c) $\frac{\sqrt{b^2 - a^2}}{b}$ 1

12. $(\sec A + \tan A)(1 - \sin A) =$
 (a) $\sec A$ (b) $\sin A$ (c) $\operatorname{cosec} A$ (d) $\cos A$
 Ans. (d) $\cos A$ 1

13. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is
 (a) 60° (b) 45° (c) 30° (d) 90°
 Ans. (d) 90° 1

14. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is
 (a) 2 units (b) π units (c) 4 units (d) 7 units
 Ans. (a) 2 units 1

15. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park is
 (a) 10 m (b) 15 m (c) 20 m (d) 24 m
 Ans. (a) 10 m 1

16. There is a green square board of side '2' unit circumscribing a red circle. Jayadev is asked to keep a dot on the abovesaid board. Find the probability that he keeps the dot on the green region.
 (a) $\frac{\pi}{4}$ (b) $\frac{4 - \pi}{4}$ (c) $\frac{\pi - 4}{4}$ (d) $\frac{4}{\pi}$
 Ans. (b) $\frac{4 - \pi}{4}$ 1

17. 2 cards of hearts and 4 cards of spades are missing from a pack of 52 cards. What is the probability of getting a black card from the remaining pack ?
 (a) $\frac{22}{52}$ (b) $\frac{22}{46}$ (c) $\frac{24}{52}$ (d) $\frac{24}{46}$
 Ans. (b) $\frac{22}{46}$ 1

18. Find the upper limit of the modal class from the given distribution.

Height (in cm)	Number of girls
Below 140	4
Below 145	11
Below 150	29
Below 155	40
Below 160	46
Below 165	51

(a) 165

(b) 160

(c) 155

(d) 150

Ans. (d) 150

1

19. DIRECTION : In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option.

Statement A (Assertion) : Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.

Statement R (Reason) : Top is obtained by fixing the plane surfaces of the hemisphere and cone together.



(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

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20. **Statement A (Assertion) :** $-5, -\frac{5}{2}, 0, \frac{5}{2}, \dots$ is in Arithmetic Progression.

Statement R (Reason) : The terms of an Arithmetic Progression cannot have both positive and negative rational numbers.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans. (c) Assertion (A) is true but reason (R) is false.

1

SECTION 'B'

Section B consists of 5 questions of 2 marks each.

21. Prove that $\sqrt{2}$ is an irrational number.

Ans. Let us assume, to the contrary, that $\sqrt{2}$ is rational.

So, we can find integers a and b such that $\sqrt{2} = \frac{a}{b}$ where a and b are coprime. 1/2

So, $b\sqrt{2} = a$

Squaring both sides,

we get $2b^2 = a^2$. 1/2

Therefore, 2 divides a^2 and so 2 divides a .

So, we can write $a = 2c$ for some integer c .

Substituting for a , we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. 1/2

This means that 2 divides b^2 , and so 2 divides b

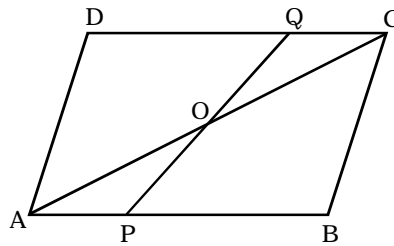
Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1. 1/2

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational.

- 22. ABCD is a parallelogram. Point P divides AB in the ratio 2 : 3 and point Q divides DC in the ratio 4 : 1. Prove that OC is half of OA.**



Ans. ABCD is a parallelogram. 1/2

$$AB = DC = a$$

Point P divides AB in the ratio 2 : 3.

$$AP = \frac{2}{5}a, \quad BP = \frac{3}{5}a$$

Point Q divides DC in the ratio 4 : 1. 1/2

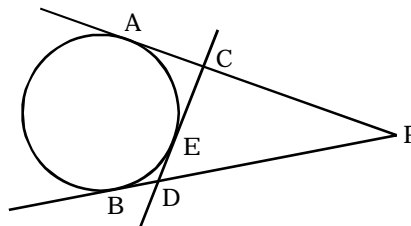
$$DQ = \frac{4}{5}a, \quad CQ = \frac{1}{5}a$$

$\triangle APO \sim \triangle CQO$ [AA similarity] 1/2

$$\frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO} \quad \text{1/2}$$

$$\frac{AO}{CO} = \frac{\frac{2}{5}a}{\frac{1}{5}a} = \frac{2}{1} \Rightarrow OC = \frac{1}{2} OA$$

- 23. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawn to intersect PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of $\triangle PCD$.**



Ans. $PA = PB$; $CA = CE$; $DE = DB$ [Tangents to a circle] 1/2

$$\begin{aligned} \text{Perimeter of } \triangle PCD &= PC + CD + PD \\ &= PC + CE + ED + PD \\ &= PC + CA + BD + PD \\ &= PA + PB \end{aligned}$$

1

$$\begin{aligned} \text{Perimeter of } \triangle PCD &= PA + PA \\ &= 2PA = 2(10) = 20 \text{ cm} \end{aligned}$$

1/2

24. If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$; $0^\circ < A + B < 90^\circ$; $A > B$, find A and B .

Ans. $\therefore \tan (A + B) = \sqrt{3} \quad \therefore A + B = 60^\circ \quad \dots(1) \quad 1/2$

$\therefore \tan (A - B) = \frac{1}{\sqrt{3}} \quad \therefore A - B = 30^\circ \quad \dots(2) \quad 1/2$

Adding (1) and (2), we get

$$2A = 90^\circ \Rightarrow A = 45^\circ \quad 1/2$$

Also (1) and (2), we get

$$2B = 30^\circ \Rightarrow B = 15^\circ \quad 1/2$$

Or

Find the value of x :

$$2 \operatorname{cosec}^2 30 + x \sin^2 60 - \frac{3}{4} \tan^2 30 = 10$$

Ans. $2 \operatorname{cosec}^2 30 + x \sin^2 60 - \frac{3}{4} \tan^2 30 = 10$

$$\Rightarrow 2(2)^2 + x \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \left(\frac{1}{\sqrt{3}}\right)^2 = 10 \quad 1$$

$$\Rightarrow 2(4) + x \left(\frac{3}{4}\right) - \frac{3}{4} \left(\frac{1}{3}\right) = 10 \quad 1/2$$

$$\Rightarrow 8 + x \left(\frac{3}{4}\right) - \left(\frac{1}{4}\right) = 10$$

$$\Rightarrow 32 + x(3) = 40$$

$$\Rightarrow 3x = 8 \Rightarrow x = \frac{8}{3} \quad 1/2$$

25. With vertices A , B and C of $\triangle ABC$ as centres, arcs are drawn with radii 14 cm and the three portions of the triangle so obtained are removed. Find the total area removed from the triangle.

Ans. Total area removed = $\frac{\angle A}{360} \pi r^2 + \frac{\angle B}{360} \pi r^2 + \frac{\angle C}{360} \pi r^2 \quad 1/2$

$$= \frac{\angle A + \angle B + \angle C}{360} \pi r^2$$

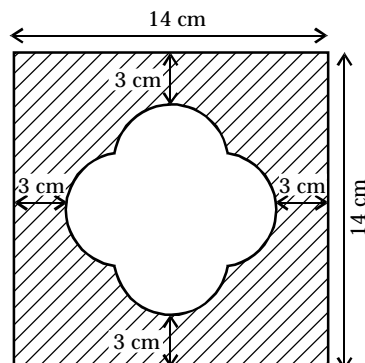
$$= \frac{180}{360} \pi r^2 \quad 1/2$$

$$= \frac{180}{360} \times \frac{22}{7} \times (14)^2 \quad 1/2$$

$$= 308 \text{ cm}^2 \quad 1/2$$

Or

Find the area of the unshaded region shown in the given figure.



Ans. The side of a square = Diameter of the semi-circle = a
 Area of the unshaded region = Area of a square of side ' a '
 $+ 4(\text{Area of a semi-circle of diameter 'a'})$ 1/2
 The horizontal/vertical extent of the white region = $14 - 3 - 3 = 8$ cm
 Radius of the semi-circle + side of a square + Radius of the semi-circle = 8 cm 1/2
 $2(\text{Radius of the semi-circle}) + \text{Side of a square} = 8$ cm
 $2a = 8$ cm $\Rightarrow a = 4$ cm 1/2
 Area of the unshaded region
 = Area of a square of side 4 cm + $4(\text{Area of a semi-circle of diameter } 4 \text{ cm})$
 $= (4)^2 + 4 \times \frac{1}{2} \pi (2)^2 = 16 + 8\pi \text{ cm}^2$ 1/2

SECTION 'C'

Section C consists of 6 questions of 3 marks each.

26. National Art Convention got registrations from students from all parts of the country, of which 60 are interested in music, 84 are interested in dance and 108 students are interested in handicrafts. For optimum cultural exchange, organisers wish to keep them in minimum number of groups such that each group consists of students interested in the same artform and the number of students in each group is the same. Find the number of students in each group. Find the number of groups in each art form. How many rooms are required if each group will be allotted a room ?

Ans. Number of students in each group subject to the given condition
 $= \text{HCF}(60, 84, 108)$ 1/2
 $\text{HCF}(60, 84, 108) = 12$ 1/2
 Number of groups in Music = $\frac{60}{12} = 5$ 1/2
 Number of groups in Dance = $\frac{84}{12} = 7$ 1/2
 Number of groups in Handicrafts = $\frac{108}{12} = 9$ 1/2
 Total number of rooms required = 21 1/2

27. If α, β are zeroes of quadratic polynomial $5x^2 + 5x + 1$, find the value of

1. $\alpha^2 + \beta^2$ 2. $\alpha^{-1} + \beta^{-1}$
Ans. $P(x) = 5x^2 + 5x + 1$ 1/2
 $\alpha + \beta = \frac{-b}{a} = \frac{-5}{5} = -1$ 1/2
 $\alpha\beta = \frac{c}{a} = \frac{1}{5}$ 1/2
 $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ 1/2
 $= (-1)^2 - 2\left(\frac{1}{5}\right)$
 $= 1 - \frac{2}{5} = \frac{3}{5}$ 1/2
 $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$ 1/2
 $= \frac{(\alpha + \beta)}{\alpha\beta} = \frac{(-1)}{\frac{1}{5}} = -5$

28. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there ?

Ans. Let the ten's and the unit's digits in the first number be x and y , respectively.

So, the original number = $10x + y$

When the digits are reversed,

x becomes the unit's digit and y becomes the ten's digit. 1/2

So, the obtain by reversing the digits = $10y + x$

According to the given condition.

$$(10x + y) + (10y + x) = 66$$

$$\text{i.e.,} \quad 11(x + y) = 66$$

$$\text{i.e.,} \quad x + y = 6 \quad \dots(1) \quad \text{1/2}$$

We are also given that the digits differ by 2,

$$\text{therefore, either } x - y = 2 \quad \dots(2) \quad \text{1/2}$$

$$\text{or } y - x = 2 \quad \dots(3) \quad \text{1/2}$$

If $x - y = 2$, then solving (1) and (2) by elimination, we get $x = 4$ and $y = 2$. 1/2

In this case, we get the number 42.

If $y - x = 2$, then solving (1) and (3) by elimination, we get $x = 2$ and $y = 4$. 1/2

In this case, we get the number 24.

Thus, there are two such numbers 42 and 24.

Or

$$\text{Solve : } \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2; \quad \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Ans. Let $\frac{1}{\sqrt{x}}$ be ' m ' and $\frac{1}{\sqrt{y}}$ be ' n ', 1/2

Then the given equations become

$$2m + 3n = 2 \quad \text{1/2}$$

$$4m - 9n = -1$$

$$(2m + 3n = 2) \times 2 \Rightarrow -4m - 6n = -4 \quad \dots(1)$$

$$4m - 9n = -1 \quad \dots(2)$$

Adding (1) and (2), we get

$$-15n = -5 \Rightarrow -n = \frac{1}{3} \quad \text{1/2}$$

Substituting $n = \frac{1}{3}$ in $2m + 3n = 2$, we get

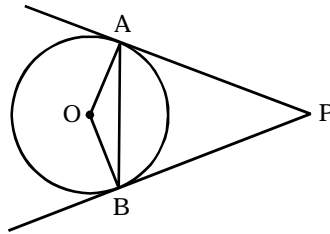
$$2m + 1 = 2$$

$$2m = 1 \quad \text{1/2}$$

$$m = \frac{1}{2}$$

$$m = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4 \text{ and } n = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9 \quad \text{1}$$

29. PA and PB are tangents drawn to a circle of centre O from an external point P. Chord AB makes an angle of 30° with the radius at the point of contact. If length of the chord is 6 cm, find the length of the tangent PA and the length of the radius OA.



Ans. $\angle OAB = 30^\circ$
 $\angle OAP = 90^\circ$ [Angle between the tangent and the radius at the point of contact] 1/2
 $\angle PAB = 90^\circ - 30^\circ = 60^\circ$
 $AP = BP$ [Tangents to a circle from an external point]
 $\angle PAB = \angle PBA$ [Angles opposite to equal sides of a triangle] 1/2

In $\triangle ABP$,
 $\angle PAB + \angle PBA + \angle APB = 180^\circ$ [Angle Sum Property]
 $60^\circ + 60^\circ + \angle APB = 180^\circ$
 $\angle APB = 60^\circ$ 1/2

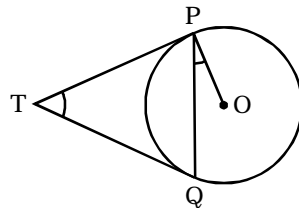
$\therefore \triangle ABP$ is an equilateral triangle, where
 $AP = BP = AB$.
 $PA = 6 \text{ cm}$ 1/2

In right $\triangle OAP$, $\angle OPA = 30^\circ$
 $\tan 30^\circ = \frac{OA}{PA}$
 $\frac{1}{\sqrt{3}} = \frac{OA}{6}$ 1/2
 $OA = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$ 1/2

Or

- Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

Ans.



Let $\angle TPQ = \theta$
 $\angle TPO = 90^\circ$ [Angle between the tangent and the radius at the point of contact] 1/2
 $\angle OPQ = 90^\circ - \theta$
 $TP = TQ$ [Tangents to a circle from an external point]
 $\angle TPQ = \angle TQP = \theta$ [Angles opposite to equal sides of a triangle] 1/2

In $\triangle PQT$,
 $\angle PQT + \angle QPT + \angle PTQ = 180^\circ$ [Angle Sum Property] 1/2
 $\theta + \theta + \angle PTQ = 180^\circ$ 1/2

$$\angle PTQ = 180^\circ - 2\theta$$

$$\angle PTQ = 2(90^\circ - \theta)$$

$$\angle PTQ = 2 \angle OPQ \quad [\text{Using (1)}]$$

1/2

1/2

30. If $1 + \sin^2\theta = 3 \sin \theta \cos \theta$, then prove that $\tan \theta = 1$ or $\frac{1}{2}$.

Ans. Given, $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$

Dividing both sides by $\cos^2 \theta$,

$$\frac{1}{\cos^2 \theta} + \tan^2 \theta = 3 \tan \theta$$

1/2

$$\sec^2 \theta + \tan^2 \theta = 3 \tan \theta$$

1/2

$$1 + \tan^2 \theta + \tan^2 \theta = 3 \tan \theta$$

1/2

$$1 + 2 \tan^2 \theta = 3 \tan \theta$$

1/2

$$2 \tan^2 \theta - 3 \tan \theta + 1 = 0$$

If $\tan^2 \theta = x$, then the equation becomes $2x^2 - 3x + 1 = 0$

$$\Rightarrow (x-1)(2x-1) = 0 \Rightarrow x = 1 \text{ or } \frac{1}{2}$$

$$\tan \theta = 1 \text{ or } \frac{1}{2}$$

1

31. The length of 40 leaves of a plant are measured correct to nearest millimetre, and the data obtained is represented in the following table.

Length (in mm)	Number of leaves
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

Find the average length of the leaves.

Ans.

Length (in mm)	Number of leaves (f)	CI	Mid x	d	fd
118 - 126	3	117.5 - 126.5	122	-27	-81
127 - 135	5	126.5 - 135.5	131	-18	-90
136 - 144	9	135.5 - 144.5	140	-9	-81
145 - 153	12	144.5 - 153.5	$a = 149$	0	0
154 - 162	5	153.5 - 162.5	158	9	45
163 - 171	4	162.5 - 171.5	167	18	72
172 - 180	2	171.5 - 180.5	176	27	54

2

$$\text{Mean} = a + \frac{\sum fd}{\sum f} = 149 + \frac{-81}{40}$$

1/2

$$= 149 - 2.025 = 146.975$$

Average length of the leaves = 146.975

1/2

SECTION 'D'

Section D consists of 4 questions of 5 marks each.

- 32. A motor boat whose speed is 18 km/h in still water takes 1 hr. more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.**

Ans. Let the speed of the stream be x km/h.

The speed of the boat upstream = $(18 - x)$ km/h and
 the speed of the boat downstream = $(18 + x)$ km/h. 1

The time taken to go upstream = $\frac{\text{Distance}}{\text{Speed}} = \frac{24}{18 - x}$ hours

the time taken to go downstream = $\frac{\text{Distance}}{\text{Speed}} = \frac{24}{18 + x}$ hours 1

According to the question,

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1 \quad 1$$

$$24(18 + x) - 24(18 - x) = (18 - x)(18 + x)$$

$$x^2 + 48x - 324 = 0$$

$$x = 6 \text{ or } -54 \quad 1$$

Since x is the speed of the stream, it cannot be negative.

Therefore, $x = 6$ gives the speed of the stream = 6 km/h. 1

Or

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans. Let the time taken by the smaller pipe to fill the tank = x hr.

Time taken by the larger pipe = $(x - 10)$ hr. 1/2

Part of the tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of the tank filled by larger pipe in 1 hour = $\frac{1}{x - 10}$ 1

The tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. 1/2

Part of the tank filled by both the pipes in 1 hour = $\frac{8}{75}$. 1/2

Therefore, $\frac{1}{x} + \frac{1}{x - 10} = \frac{8}{75}$ 1/2

$$8x^2 - 230x + 750 = 0$$

$$x = 25, \frac{30}{8} \quad 1$$

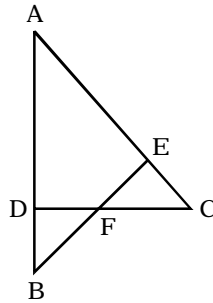
Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours, as the time taken by the larger pipe will become negative, which is logically not possible. 1/2

Therefore, the time taken individually by the smaller pipe and the larger pipe will be 25 and $25 - 10 = 15$ hours, respectively. 1/2

33. (a) State and prove Basic Proportionality theorem.

(b) In the given figure $\angle CEF = \angle CFE$. F is the midpoint of DC.

Prove that $\frac{AB}{BD} = \frac{AE}{FD}$.



Ans. (a) Statement — $\frac{1}{2}$
 Given and To Prove — $\frac{1}{2}$
 Figure and Construction — $\frac{1}{2}$
 Proof — $1\frac{1}{2}$
 (b) Draw $DG \parallel BE$

In $\triangle ABE$, $\frac{AB}{BD} = \frac{AE}{GE}$ [BPT]
 $CF = FD$ [F is the midpoint of DC] ... (j)

In $\triangle CDG$, $\frac{DF}{CF} = \frac{GE}{CE} = 1$ [Mid point theorem]

$GE = CE$... (i) 1/2

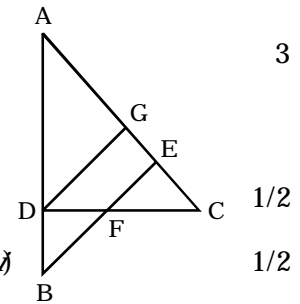
$\triangle CEF = \triangle CFE$ [Given]

$CF = CE$ [Sides opposite to equal angles] ... (ii) 1/2

From (i) and (ii), $CF = GE$... (i)

From (j) and (i), $GE = FD$

$$\therefore \frac{AB}{BD} = \frac{AE}{GE} \Rightarrow \frac{AB}{BD} = \frac{AE}{FD}$$



34. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm ?

What should be the speed of water if the rise in water level is to be attained in 1 hour ?

Ans. Length of the pond, $l = 50$ m, width of the pond, $b = 44$ m

$$\text{Water level is to rise by, } h = 21 \text{ cm} = \frac{21}{100} \text{ m}$$

$$\text{Volume of water in the pond} = lbh = 50 \times 44 \times \frac{21}{100} \text{ m}^3 = 462 \text{ m}^3 \quad 1$$

Diameter of the pipe = 14 cm

$$\text{Radius of the pipe, } r = 7 \text{ cm} = \frac{7}{100} \text{ m}$$

Area of cross-section of pipe = πr^2

$$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} = \frac{154}{10000} \text{ m}^2 \quad 1$$

Rate at which the water is flowing through the pipe,

$$h = 15 \text{ km/h} = 15000 \text{ m/h} \quad 1/2$$

Volume of water flowing in 1 hour = Area of cross-section of pipe \times height of water coming

out of pipe 1/2

$$= \left(\frac{154}{10000} \times 15000 \right) \text{m}^3 \quad 1$$

$$\text{Time required to fill the pond} = \frac{\text{Volume of the pond}}{\text{Volume of water flowing in 1 hour}} \quad 1$$

$$= \frac{462 \times 10000}{154 \times 15000} = 2 \text{ hours}$$

Speed of water if the rise in water level is to be attained in 1 hour = 30 km/h

Or

A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent, keeping a provision of 26 m² of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 500 per m².

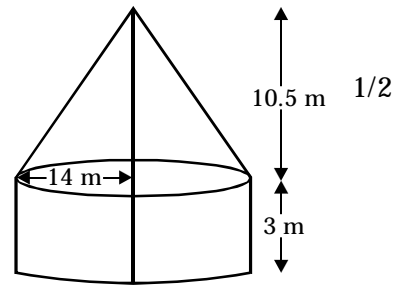
Ans. Radius of the cylindrical tent (r) = 14 m

Total height of the tent = 13.5 m

Height of the cylinder = 3 m

Height of the conical part = 10.5 m

$$\begin{aligned} \text{Slant height of the cone } (l) &= \sqrt{h^2 + r^2} \\ &= \sqrt{(10.5)^2 + (14)^2} \\ &= \sqrt{110.25 + 196} \\ &= \sqrt{306.25} = 17.5 \text{ m} \end{aligned}$$



Curved surface area of cylindrical portion

$$\begin{aligned} &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 14 \times 3 \\ &= 264 \text{ m}^2 \end{aligned} \quad 1$$

Curved surface area of conical portion = πrl

$$\begin{aligned} &= \frac{22}{7} \times 14 \times 17.5 \\ &= 770 \text{ m}^2 \end{aligned} \quad 1$$

$$\begin{aligned} \text{Total curved surface area} &= 264 \text{ m}^2 + 770 \text{ m}^2 \\ &= 1034 \text{ m}^2 \end{aligned} \quad 1/2$$

$$\text{Provision for stitching and wastage} = 26 \text{ m}^2 \quad 1/2$$

$$\text{Area of canvas to be purchased} = 1060 \text{ m}^2$$

$$\text{Cost of canvas} = \text{Rate} \times \text{Surface area}$$

$$= 500 \times 1060$$

$$= ₹ 5,30,000/- \quad 1/2$$

35. The median of the following data is 50. Find the values of 'p' and 'q', if the sum

of all frequencies is 90. Also find the mode.

Marks obtained	Number of students
20 – 30	p
30 – 40	15
40 – 50	25
50 – 60	20
60 – 70	q
70 – 80	8
80 – 90	10

Ans.

Marks obtained	Number of students	Cumulative frequency
20 – 30	p	p
30 – 40	15	$p + 15$
40 – 50	25	$p + 40$
50 – 60	20	$p + 60$
60 – 70	q	$p + q + 60$
70 – 80	8	$p + q + 68$
80 – 90	10	$p + q + 78$
	90	

2

$$p + q + 78 = 90$$

$$p + q = 12$$

$$\text{Median} = (l) + \frac{\frac{n}{2} - cf}{f} \cdot h$$

$$50 = 50 + \frac{45 - (p + 40)}{20} \cdot 10$$

1/2

$$\frac{45 - (p + 40)}{20} \cdot 10 = 0$$

1/2

$$45 - (p + 40) = 0$$

1/2

$$p = 5$$

1/2

$$5 + q = 12$$

$$q = 7$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \cdot h$$

1

$$= 40 + \frac{25 - 15}{2(25) - 15 - 20} \cdot 10$$

$$= 40 + \frac{100}{15} = 40 + 6.67 = 46.67$$

SECTION 'E'

36. Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of 18.86m at the Asian Grand Prix in 2017 is the biggest distance for an Indian female athlete.



Keeping her as a role model, Sanjitha is determined to earn gold in Olympics one day.

Initially her throw reached 7.56 m only. Being an athlete in school, she regularly practiced both in the mornings and in the evenings and was able to improve the distance by 9cm every week.

During the special camp for 15 days, she started with 40 throws and every day kept increasing the number of throws by 12 to achieve this remarkable progress.

(i) How many throws Sanjitha practiced on 11th day of the camp ?

(ii) What would be Sanjitha's throw distance at the end of 6 months ?

Or

When will she be able to achieve a throw of 11.16 m ?

(iii) How many throws did she do during the entire camp of 15 days ?

Ans. (i) Number of throws during camp, $a = 40$; $d = 12$ 1

$$\begin{aligned} t_{11} &= a + 10d \\ &= 40 + 10 \times 12 \\ &= 160 \text{ throws} \end{aligned}$$

(ii) $a = 7.56$ m; $d = 9$ cm = 0.09 m 1/2

$$\begin{aligned} n &= 6 \text{ weeks} && 1/2 \\ t_n &= a + (n - 1) d && 1/2 \\ &= 7.56 + 6 (0.09) \\ &= 7.56 + 0.54 \end{aligned}$$

Sanjitha's throw distance at the end of 6 weeks = 8.1 m 1/2

Or

$a = 7.56$ m; $d = 9$ cm = 0.09 m 1/2

$$\begin{aligned} t_n &= 11.16 \text{ m} \\ t_n &= a + (n - 1) d && 1/2 \\ 11.16 &= 7.56 + (n - 1) (0.09) && 1/2 \\ 3.6 &= (n - 1) (0.09) \end{aligned}$$

$$n - 1 = \frac{3.6}{0.09} = 40$$

$$n = 41 \quad 1/2$$

Sanjitha's will be able to throw 11.16 m in 41 weeks.

(iii) $a = 40$; $d = 12$; $n = 15$ 1/2

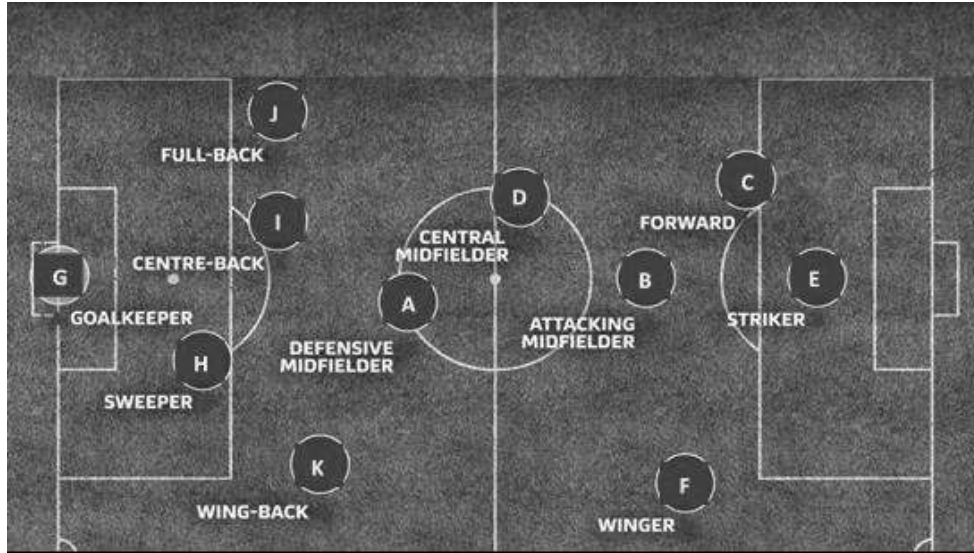
$$S_n = \frac{n}{2} [2a + (n - 1) d]$$

$$S_n = \frac{15}{2} [2(40) + (15 - 1) (12)]$$

$$= \frac{15}{2} [80 + 168]$$

$$= \frac{15}{2} [248] = 1860 \text{ throws} \quad 1/2$$

37. Tharunya was thrilled to know that the football tournament is fixed with a monthly timeframe from 20th July to 20th August 2023 and for the first time in the FIFA Women's World Cup's history, two nations host in 10 venues. Her father felt that the game can be better understood if the position of players is represented as points on a coordinate plane.



- (i) At an instance, the midfielders and forward formed a parallelogram. Find the position of the central midfielder (D) if the position of other players who formed the parallelogram are : A(1, 2), B(4,3) and C(6,6).
- (ii) Check if the Goal keeper G(-3, 5), Sweeper H(3, 1) and Wing-back K(0, 3) fall on a same straight line.

Or

Check if the Full-back J(5, -3) and centre-back I (-4,6) are equidistant from forward C(0, 1) and if C is the mid-point of IJ.

- (iii) If defensive midfielder A(1, 4), Attacking midfielder B(2, -3) and Striker E(a, b) lie on the same straight line and B is equidistant from A and E, find the position of E.

Ans. (i) Let D be (a, b), then

$$\text{Mid point of AC} = \text{Midpoint of BD} \quad 1/2$$

$$\left(\frac{1+6}{2}, \frac{2+6}{2} \right) = \left(\frac{4+a}{2}, \frac{3+b}{2} \right)$$

$$\begin{aligned} 4+a &= 7 & 3+b &= 8 \\ a &= 3 & b &= 5 \end{aligned}$$

Central midfielder is at (3, 5) 1/2

(ii) $GH = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ 1/2

$$GK = \sqrt{(0+3)^2 + (3-5)^2} = \sqrt{9+4} = \sqrt{13} \quad 1/2$$

$$HK = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{9+4} = \sqrt{13} \quad 1/2$$

$GK + HK = GH \Rightarrow G, H$ and K lies on a same straight line.

Or

$$CI = \sqrt{(0-5)^2 + (1+3)^2} = \sqrt{25+16} = \sqrt{41} \quad 1/2$$

$$CI = \sqrt{(0+4)^2 + (1-6)^2} = \sqrt{16+25} = \sqrt{41} \quad 1/2$$

Full-back J(5, - 3) and centre-back I (- 4, 6) are equidistant from forward C (0,1)

$$\text{Mid-point of IJ} = \left(\frac{5 - 4}{2}, \frac{-3 + 6}{2} \right) = \left(\frac{1}{2}, \frac{3}{2} \right) \quad 1/2$$

C is NOT the mid-point of IJ. 1/2

(iii) A,B and E lie on the same straight line and B is equidistant from A and E

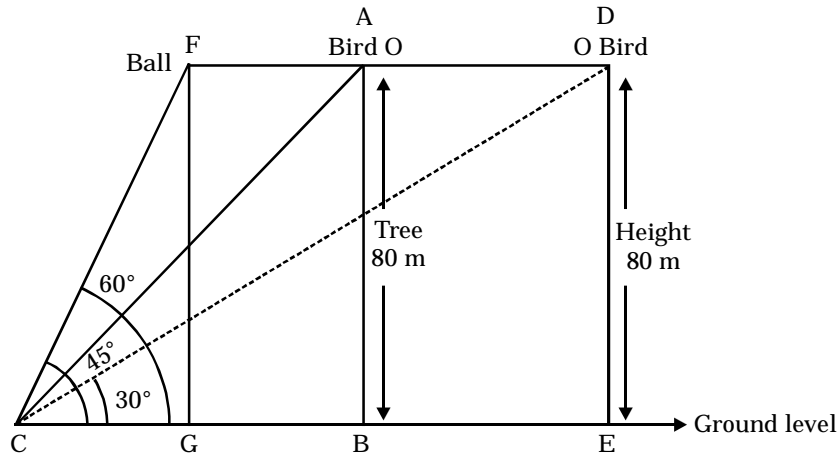
⇒ B is the mid-point of AE

$$\left(\frac{1 + a}{2}, \frac{4 + b}{2} \right) = (2, -3) \quad 1/2$$

$$1 + a = 4; a = 3. 4 + b = -6; b = -10 \text{ E is } (3, -10) \quad 1/2$$

38. One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on a nearby tree of height 80 m. He observed a bird on the tree at an angle of elevation of 45°.

When a sixer was hit, a ball flew through the tree frightening the bird to fly away. In 2 seconds, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.



(i) At what distance from the foot of the tree was he observing the bird sitting on the tree ?

(ii) How far did the bird fly in the mentioned time ?

Or

After hitting the tree, how far did the ball travel in the sky when Kaushik saw the ball?

(iii) What is the speed of the bird in m/min if it had flown $20(\sqrt{3} + 1)$ m ?

Ans. (i) $\tan 45^\circ = \frac{80}{CB} \Rightarrow CB = 80 \text{ m}$ 1

(ii) $\tan 30^\circ = \frac{80}{CE}$ 1/2+1/2

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{CE}$$

$$\Rightarrow CE = 80\sqrt{3}$$

$$\text{Distance the bird flew} = AD = BE = CE - CB = 80\sqrt{3} - 80 = 80(\sqrt{3} - 1) \text{ m}$$

Or

$$\begin{aligned}\tan 60^\circ &= \frac{80}{CG} \\ \Rightarrow \sqrt{3} &= \frac{80}{CG} \\ \Rightarrow CG &= \frac{80}{\sqrt{3}}\end{aligned}$$

Distance the ball travelled after hitting the tree
= FA = GB = CB - CG

$$GB = 80 - \frac{80}{\sqrt{3}} = 80 \left(1 - \frac{1}{\sqrt{3}}\right) \text{ m}$$

$$\begin{aligned}(\text{iii}) \text{ Speed of the bird} &= \frac{\text{Distance}}{\text{Time taken}} = \frac{20(\sqrt{3} + 1)}{2} \text{ m/sec} \\ &= \frac{20(\sqrt{3} + 1)}{2} \times 60 \text{ m/min} = 600 (\sqrt{3} + 1) \text{ m/min}\end{aligned}$$