Unit I. Number Systems

Real Numbers

CHAPTER 1

SYLLABUS

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$.

In this chapter you will study

- **Fundamental Theorem of Arithmetic and its Applications**
- **Irrational Number**

MIND-MAP

By Prime Factorisation Method

HCF

Step 1 : Find prime factorisations of the given numbers.

Step 2 : Identify common prime factors.

Step 3 : Find the product of all the common prime factors, using each common factor the *least number* of times it appears in the prime factorisation of any of the given numbers. The product so obtained is the required HCF.

LCM

Step 1 : Find the prime factorisations of the given numbers.

Step 2 : Find product of all different prime factors of the numbers, using each prime factor the *greatest number* of times it appears in the prime factorisation of any of the numbers. The product so obtained is the required LCM.

Fundamental Theorem of Arithmetic

- Every composite number can be expressed as a product of primes and this factorisation is unique, except for the order in which these prime factors occur.
- If p is a prime and p divides a^2, then p divides a also, where a is a positive integer.
- It can be proved that $\sqrt{p}$ is an irrational number, where p is any prime.

Properties of HCF and LCM

(*a*) HCF of given numbers is always a factor of their LCM.

(*b*) If HCF of two numbers is 1, then they are called co-prime.

(*c*) LCM of two co-prime is equal to their product.

(*d*) If a and b are any two numbers, then $a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b)$

(*e*) If a, b and c are three numbers, then $a \times b \times c \neq \text{HCF}(a, b, c) \times \text{LCM}(a, b, c)$

Objective Type Questions (**1** Mark)
Short Answer Type Questions-I (**2** Marks)
Short Answer Type Questions-II (**3** Marks)
Long Answer Type Questions (**4** Marks)

Typology of Questions Ⓡ Remembering; Ⓐ Applying; Ⓤ Understanding; A/E Analysing, Evaluating and Creating

Thick Letters (Bold Face) for Standard questions
Thin Letters (Normal Face) for Basic questions

Topic 1. Fundamental Theorem of Arithmetic and its Applications

Revision Notes

Every composite number can be expressed (factorised) as *a* product of primes and this expression (Factorisation) is unique, apart from the order in which the prime factors occur.

This theorem says that given any composite number, it can be factorised as a product of prime numbers in a unique way except for the order in which the primes occur.

Once we have decided that the order will be ascending, then the way the number is factorised, is unique. For example, factors of 140 may be

$$140 = 2 \times 2 \times 5 \times 7$$
$$140 = 2 \times 5 \times 2 \times 7$$
$$140 = 5 \times 2 \times 2 \times 7$$
$$140 = 5 \times 7 \times 2 \times 2, \text{ etc.}$$

Here, factors are same but orders of primes in which they have been put are different. Now, we say to write the factors in ascending order. Then, we write it as $140 = 2 \times 2 \times 5 \times 7$ which is unique. No other possibility of factorisation is there.

This theorem is used to find the HCF and LCM of two or more numbers by prime factorisation method. In this method :

***Step* 1 :** Factorise each of the given numbers and put them in the form of power of each factor.

***Step* 2 :** Write the HCF as the product of the smallest power of each common prime factor of the numbers.

***Step* 3 :** Write the LCM as the product of the greatest power of each prime factor involved in the numbers.

***Example* 1 :** Determine the values of p and q so that the prime factorisation of 2520 is expressible as $2^3 \times 3^p \times q \times 7$.

Sol. $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2^3 \times 3^2 \times 5 \times 7$

Comparing with $2^3 \times 3^p \times q \times 7$, we get

$p = 2$ and $q = 5$.

2	2520
2	1260
2	630
3	315
3	105
5	35
7	7
	1

***Example* 2 :** Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pens and notepads.

Sol. Required number of equal pens and notepads for least number of pack of each type is the LCM of 8 and 12.

Now, $8 = 2 \times 2 \times 2 = 2^3$

and $12 = 2 \times 2 \times 3 = 2^2 \times 3$

So, $\text{LCM} = 2^3 \times 3 = 24.$

Hence, least number of packs of pens $= \frac{24}{12} = 3.$

and least number of packs of notepads $= \frac{24}{12} = 2.$

***Example* 3 :** A rectangular courtyard is 18 m 72 cm long and 13 m 20 cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.

Sol. Length = 18 m 72 cm = 1872 cm

Breadth = 13 m 20 cm = 1320 cm

For **least number** of tiles, the tile should be of **maximum size.**

So, we first find the HCF of 1872 and 1320.

$$1872 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 13$$
$$1320 = 2 \times 2 \times 2 \times 3 \times 5 \times 11$$

So, HCF (1872, 1320)

$= 2 \times 2 \times 2 \times 3 = 24.$

Hence, maximum size of the tile must be the square tile of side 24 cm.

Hence, required least number of tiles

$$= \frac{1872 \times 1320}{24 \times 24} = 78 \times 55 = 4290.$$

EXERCISE 1.1

Competency Based Questions

Multiple Choice Questions

® 1. If '*p*' and '*q*' are natural numbers and '*p*' is the multiple of '*q*', then what is the HCF of '*p*' and '*q*' ? ***(CBSE 30-2-1, 2023)***

(*a*) pq (*b*) p (*c*) q (*d*) $p + q$

Sol. (*c*) q

Explanation : This is because the HCF is the greatest common divisor of the numbers, and since '*q*' is a factor of '*p*', it is the greatest common divisor of '*p*' and '*q*'.

Note **: Thick Letters (Bold Face) for Standard Questions /** Thin Letters (Normal Face) for Basic Questions

Ⓤ 2. **If '*n*' is a natural number, then which of the following numbers ends with zero ?**

(a) $(3 \times 2)^n$ (b) $(2 \times 5)^n$

(c) $(6 \times 2)^n$ (d) $(5 \times 3)^n$

(CBSE 30-2-3, 2023)

Sol. (b) $(2 \times 5)^n$

Explanation : This is because any number that includes both 2 and 5 as prime factors will end with zero when raised to any power 'n'.

Ⓡ 3. **The ratio of HCF to LCM of the least composite number and the least prime number is :** ***(CBSE 30-4-1, 2023)***

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

Sol. (a) 1 : 2

Explanation : The least composite number is 4, which can be factorized as 2^2, and the least prime number is 2. The HCF of 4 and 2 is 2, and the LCM of 4 and 2 is 4. Therefore, the ratio of HCF to LCM is 2 : 4

Simplifying this ratio, we get : 1 : 2.

Ⓐ 4. **The LCM of smallest 2-digit number and smallest composite number is**

(a) 12 (b) 4 (c) 20 (d) 40

(CBSE 30-6-2, 2023)

Sol. (c) 20

Explanation : The smallest 2-digit number is 10, and the smallest composite number is 4. The LCM of 10 and 4 can be calculated as follows :

$$10 = 2 \times 5$$
$$4 = 2 \times 2$$

LCM (10, 4) = $2 \times 2 \times 5 = 20$.

Ⓡ 5. **In a car racing competition, the time taken by two racing cars A and B to complete 1 round of the track is 30 minutes and *p* minutes respectively. If the cars meet again at the starting point for the first time after 90 minutes and the HCF (30, *p*) = 15, then the value of *p* is**

(a) 45 minutes (b) 60 minutes

(c) 75 minutes (d)180 minutes

(CBSE Addl. Practice, 2023-24)

Ans. (a) 45 minutes

Explanation : The time taken by the two cars to meet again at the starting point for the first time is given by the LCM of their individual times. We are given that the LCM of 30 and p is 90, and the HCF of 30 and p is 15.

We can use the relationship between LCM and HCF, which states that LCM(a, b) × HCF (a, b) = $a \times b$.

So, in this case, we have :

$$\text{LCM}(30, p) \times \text{HCF}(30, p) = 30 \times p$$
$$90 \times 15 = 30 \times p$$
$$1350 = 30p$$
$$p = \frac{1350}{30}$$
$$p = 45$$

Ⓐ 6. The prime factorisation of natural number 288 is

(a) $2^4 \times 3^3$ (b) $2^4 \times 3^2$

(c) $2^5 \times 3^2$ (d) $2^5 \times 3^1$

(CBSE 430-1-1, 2023)

Sol. (c) $2^5 \times 3^2$

Explanation : The prime factorization of the natural number 288 is given by :

$288 = 2^5 \times 3^2$

Ⓡ 7. The prime factorisation of the natural number 432 is ***(CBSE 430-1-2, 2023)***

(a) $2^3 \times 3^4$ (b) $2^4 \times 3^3$

(c) $2^3 \times 3^3$ (d) $2^4 \times 3^4$

Ans. (b) $2^4 \times 3^3$

Explanation : The prime factorization of the natural number 432 is given by :

$432 = 2^4 \times 3^3$

Ⓤ 8. The prime factorisation of 1728 is

(a) $2^5 \times 3^3$ (b) $2^5 \times 3^4$

(c) $2^6 \times 3^3$ (d) $2^6 \times 3^2$

(CBSE 430-1-3, 2023)

Sol. (c) $2^6 \times 3^3$

Explanation : The prime factorization of the natural number 1728 is given by :

$1728 = 2^6 \times 3^3$

Ⓐ 9. If the HCF of 360 and 64 is 8, then their LCM is :

(a) 2480 (b) 2780 (c) 512 (d) 2880

(CBSE 430-2-1, 2023)

Sol. (d) 2880

Explanation : We can use the relationship between LCM and HCF, which states that LCM (a, b) × HCF (a, b) = $a \times b$.

We are given that the HCF of 360 and 64 is 8. So, we can write :

$$360 = 8 \times 45$$
$$64 = 8 \times 8$$

Now, we can find the LCM of 360 and 64 as follows :

LCM (360, 64) = (360 × 64) / HCF (360, 64)

LCM (360, 64) = (360 × 64) / 8

LCM (360, 64) = 2880

Ⓤ **10.** If the HCF of 72 and 234 is 18, then the LCM (72, 234) is :

(*a*) 936 (*b*) 836 (*c*) 324 (*d*) 234

(*CBSE 430-2-2, 2023*)

Sol. (*a*) 936

Explanation : We can use the relationship between LCM and HCF, which states that LCM $(a, b) \times$ HCF $(a, b) = a \times b$.

We are given that the HCF of 72 and 234 is 18. So, we can write :

72 = 18 × 4

234 = 18 × 13

Now, we can find the LCM of 72 and 234 as follows :

LCM (72, 234) = (72 × 234) / HCF (72, 234)

LCM (72, 234) = (72 × 234) / 18

LCM (72, 234) = 936

Assertion Reason Type

Following questions consist of two statements — Assertion (A) and Reason (R). Answer these questions, selecting appropriate option given below :

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

(*b*) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*

(*c*) *(A) is true but (R) is false.*

(*d*) *(R) is true but (A) is false.*

Ⓐ **11. (A) :** The number 5^n cannot end with the digit 0, where n is a natural number.

(R) : Prime factorisation of 5 has only two factors, 1 and 5. **(*CBSE 30-5-1, 2023*)**

Ans. (*c*)

Ⓤ **12. (A) :** If HCF$(a, 8) = 4$ and LCM$(a, 8) = 24$, then a is equal to 12.

(R) : Product of two numbers is equal to product of their HCF and LCM.

Ans. (*a*)

Very Short Answer Type Questions

Ⓐ **13.** Find the largest number which divides 23 and 33 leaving remainders 2 and 5 respectively.

Sol. Required number will be the HCF of

23 – 2 = 21 and 33 – 5 = 28

i.e., HCF (21, 28)

Now, 28 = 2 × 2 × 7

21 = 3 × 7

So, HCF (28, 21) = 7.

Hence, the required number is 7.

Ⓤ **14.** Explain whether 3 × 12 × 101 + 4 is a prime number or a composite number.

Sol. We have :

$$3 \times 12 \times 101 + 4 = 4(3 \times 3 \times 101 + 1) = 4(909 + 1) = 4 \times 910$$

Since the given number has a factor 4 other than 1 and the number itself. So, the number is a composite number.

SHORT ANSWER TYPE QUESTIONS - I

Ⓐ **15. Find the greatest 3-digit number which is divisible by 18, 24 and 36.**

(*CBSE 30-5-3, 2023*)

Sol. LCM of 18, 24, 36 is 72.

Required greatest 3-digit number

= 13 × 72 = 936.

Ⓤ **16. Using prime factorisation, find HCF and LCM of 96 and 120.**

(*CBSE 30-5-1, 2023*)

Sol. $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$

HCF $= 2^3 \times 3 = 24$

LCM $= 2^5 \times 3 \times 5 = 480$

Ⓐ **17. Show that 6^n can not end with digit 0 for any natural number '*n*'.**

(NCERT TBQ) (*CBSE 30-6-1, 2023*)

Sol. If 6^n ends with digit 0, it would be divisible by 5. So, prime factorization of 6^n would contain 5. But $6^n = (2 \times 3)^n$, the only prime factorization of 6^n are 2 and 3 as per fundamental theorem of Arithmetic. There is no other prime in the factorization of 6^n. So, there is no natural number n for which 6^n ends with digit zero.

Ⓤ **18. A forester wants to plant 66 apple trees, 88 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also, he wants to make distinct rows of the trees (only one type of tree in one row). Find the minimum number of rows required.** **(*CBSE Addl. Practice, 2023-24*)**

***Note* : Thick Letters (Bold Face) for Standard Questions** / Thin Letters (Normal Face) for Basic Questions

Sol.

$$66 = 2 \times 3 \times 11$$
$$88 = 2^3 \times 11$$
$$110 = 2 \times 5 \times 11$$
$$\text{HCF} = 2 \times 11 = 22$$
$$\text{Total trees} = 264$$
$$\therefore \text{ Total number of rows } = \frac{264}{22} = 12.$$

Ⓤ 19. **Find the smallest pair of 4-digit numbers such that the difference between them is 303 and their HCF is 101. Show your steps.** **(*CBSE Addl. Practice, 2022-23*)**

Ans. Find that the two numbers are of the form $101p$ and $101q$ where $p > q$ and p and q are co-prime to each other.

Use the given information and write :

$$101p - 101q = 303$$
$$\Rightarrow \quad 101(p - q) = 303$$
$$\Rightarrow \quad p - q = 3$$
$$\Rightarrow \quad p = q + 3$$

Identify that the smallest 4-digit number can be found when q and p are 10 and 13 respectively. Find the two numbers as 1010 and 1313.

Short Answer Type Questions - II

Ⓤ 20. **By prime factorisation, find the LCM of the numbers 18180 and 7575. Also, find the HCF of the two numbers.**

(*CBSE 30-2-1, 2023*)

Sol.

$$18180 = 2^2 \times 3^2 \times 5 \times 101$$
$$7575 = 3 \times 5^2 \times 101$$
$$\text{LCM} = 2^2 \times 3^2 \times 5^2 \times 101 = 90900$$
$$\text{HCF} = 3 \times 5 \times 101 = 1515$$

Ⓐ 21. **Three bells ring at intervals of 6, 12 and 18 minutes. If all the three bells rang at 6 a.m., when will they ring together again ?** **(*CBSE 30-2-1, 2023*)**

Sol. LCM of 6, 12, 18 = 36

So, all the three bells ring will together after 36 minutes at 6 : 36 a.m.

Ⓡ 22. **Prove that 4^n can never end with digit 0, where n is a natural number.**

(*CBSE 30-2-2, 2023*)

Sol. If the number 4^n, for any n, were to end with digit zero, it would be divisible by 5. So, the prime factorization of 4^n should contain the prime factor 5.

But in $4^n = (2 \times 2)^n = 2^{2n}$, the only prime factor is 2.

$\therefore$ By fundamental theorem of arithmetic, there is no natural number n for which 4^n ends with digit zero.

Ⓤ 23. **The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change together next ?** **(*CBSE 30-5-1, 2023*)**

Sol. LCM = 432

i.e., $\frac{432}{60}$ = 7 min 12 sec.

$\Rightarrow$ Traffic lights will change simultaneously again at 7 : 7 : 12 a.m.

Ⓐ 24. **Find the HCF and LCM of 26, 65 and 117, using prime factorisation.**

(*CBSE 30-6-1, 2023*)

Sol.

$$26 = 13 \times 2$$
$$65 = 13 \times 5$$
$$117 = 13 \times 3 \times 3$$
$$\therefore \quad \text{HCF} = 13$$
$$\text{LCM} = 13 \times 2 \times 3 \times 5 \times 3 = 1170$$

Long Answer Type Questions

(A/E) 25. A trader with a basket of eggs finds that if he sells 3 eggs at a time, there is only 1 egg left. If he sells 4 eggs at a time, there is again 1 egg left. However, if the trader sells 7 eggs at a time, there are no eggs left. If the capacity of the basket is 100 eggs, find how many eggs are there in the basket. Explain with reasoning.

(2016-0IEZW9R, 7KVMNQP)

HINT: Number of eggs is a multiple of 7 and (multiple of LCM of 3 and 4) + 1.

Sol. LCM of 3 and 4 = 3 × 4 = 12.

The capacity of the basket is 100.

So, there may be 12 × 8 + 1 = 96 + 1 = 97 eggs.

But, 97 is not divisible by 7.

Again, there may be

$$12 \times 7 + 1 = 85$$
or $$12 \times 6 + 1 = 73$$
or $$12 \times 5 + 1 = 61$$
or $$12 \times 4 + 1 = 49 \text{ eggs.}$$

Among these numbers 85, 73 and 61 are not divisible by 7. But 49 is divisible by 7. So, the number of eggs in the basket is 49.

A/E **26.** A boy with a collection of marbles realises that if he makes a group of 5 or 6 marbles at a time, there are always 2 marbles left. Can you explain why the boy cannot have prime number of marbles ? (2016-7WDLUNH, 7TQCVA7)

Sol. The LCM of 5 and 6 = 5 × 6 = 30.

So, the number of marbles that the boy can have is $30n + 2$

So, $30n + 2 = 2\,(15n + 1)$

which has more than two factors namely 2, $(15n + 1)$ and 1. So, it cannot be a prime number.

A/E **27. The sum of LCM and HCF of two numbers is 7380. If the LCM of these numbers is 7340 more than their HCF, find the product of the two numbers.**

(2016-PZ8S1LO, SQ1P44V)

Sol. Let LCM = x and HCF = y.

Then, $x + y = 7380$ ...(1)

Also, $x = y + 7340$

$\Rightarrow$ $x - y = 7340$ ...(2)

Adding (1) and (2), we get

$$x + y = 7380$$
$$x - y = 7340$$
$$\Rightarrow \quad 2x = 14720$$
$$\Rightarrow \quad x = \frac{14720}{2} = 7360.$$

From eqn. (1),

$$7360 + y = 7380$$
$$\Rightarrow \quad y = 7380 - 7360 = 20.$$

So, product of the two numbers

= HCF × LCM

= 20 × 7360 = 147200.

Practice Exercise

Objective Type Questions

I. *Multiple Choice Questions*

Ⓡ 1. (HCF × LCM) for the numbers 70 and 40 is : ***(CBSE 430-4-1, 2023)***

(*a*) 10 (*b*) 280

(*c*) 2800 (*d*) 70

Ⓐ 2. (HCF × LCM) for the numbers 30 and 70 is : ***(CBSE 430-4-2, 2023)***

(*a*) 2100 (*b*) 21

(*c*) 210 (*d*) 70

Ⓤ 3. LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is :

(*a*) 40 (*b*) 560

(*c*) 1680 (*d*) 1120

(CBSE 430-5-1, 2023)

II. ***Assertion Reason Type***

Following questions consist of two statements — Assertion (A) and Reason (R). Answer these questions, selecting appropriate option given below :

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

(*b*) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*

(*c*) *(A) is true but (R) is false.*

(*d*) *(R) is true but (A) is false.*

Ⓡ **4. A number q is prime factorised as 32 × 72 × b, where b is a prime number other than 3 and 7.**

Based on the above information, two statements are given below — one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

(A) : q is definitely an odd number.

(R) : $3^2 \times 7^2$ is an odd number.

(CBSE Addl. Practice, 2023-24)

III. ***Very Short Answer Type Questions***

Ⓤ **5.** If a and b are two positive integers such that $a = 14b$. Find the HCF of a and b.

Ⓐ **6.** Find the least positive integer which is divisible by first five natural numbers.

Short Answer Type Questions - I

Ⓤ 7. **Find the least number which when divided by 12, 16 and 24 leaves remainder 7 in each case.**

(CBSE 30-1-2, 2023)

Ⓡ 8. **Find the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively.** ***(CBSE 30-1-1, 2023)***

Ⓤ 9. **Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers ?** ***(CBSE 30-4-1, 2023)***

Ⓤ 10. Find the LCM and HCF of 92 and 510, using prime factorisation. **(NCERT TBQ)**

(CBSE 430-1-1, 2023)

Note **: Thick Letters (Bold Face) for Standard Questions** / Thin Letters (Normal Face) for Basic Questions

Ⓐ 11. Find the HCF of the numbers 540 and 630, using prime factorization method.
(*CBSE 430-2-1, 2023*)

Ⓡ 12. Show that $(15)^n$ cannot end with the digit 0 for any natural number 'n'.
(*CBSE 430-2-1, 2023*)

Ⓐ **13.** Find LCM of 576 and 512 by prime factorization. **(*CBSE 430-5-1, 2023*)**

Ⓡ **14.** Find HCF of 660 and 704 by prime factorization. **(*CBSE 430-5-2, 2023*)**

Ⓤ **15.** Find LCM of 480 and 256 using prime factorization. **(*CBSE 430-5-3, 2023*)**

Ⓐ **16.** Show that 8^n can never end with the digit 0 for any natural number n.
(*CBSE Comptt., 2023*)

Short Answer Type Questions - II

Ⓐ 17. Prime factorisation of three numbers A, B and C is given below :

$A = (2^r \times 3^p \times 5^q)$

$B = (2^p \times 3^r \times 5^p)$

$C = (2^q \times 3^q \times 5^p)$ such that, $p < q < r$ and p, q, and r are natural numbers.

- The largest number that divides A, B and C without leaving a remainder is 30.
- The smallest number that leaves a remainder of 2 when divided by each of A, B and C is 5402.

Find A, B and C. Show your work.
(*CBSE Addl. Practice, 2023-24*)

Ⓡ 18. **The LCM of 6^4, 8^2 and k is 12^4 where k is a positive integer. Find the smallest value of k. Show your steps.**
(*CBSE Addl. Practice, 2022-23*)

A/E 19. Show that 9^n can't end with 2 for any integer n.

[*Hint :* Prime factor of 9 = 3 × 3]

A/E 20. Check whether 15^n can end with the digit zero (0) for any natural number n. (2016-LO3AGID, EGP42MY; 2015-P9KKWON; 2012-63; 2011-560027; 2010-1040119-B1)

[*Hint :* Prime factors 15 = 3 × 5]

A/E 21. Prove that for any positive integer n, $n^3 - n$ is divisible by 6.
(2016-S16MA2I; 2015-PRX7O9M; 2014-9SY5YW6; 2011-560022)

[*Hint :* $n^3 - n = n(n-1)(n+1)$]

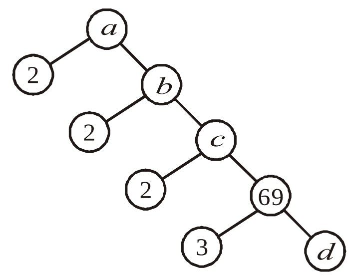
Ⓐ 22. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

[*Hint :* Use HCF]

Long Answer Type Questions

A/E 23. State Fundamental theorem of Arithmetic.
Is it possible for the HCF and LCM of two numbers to be 18 and 378 respectively. Justify your answer.
(2016-7AVCBK4, 8VEBXZZ)

Ⓤ 24. Find the values of a, b, c and d in the factor tree.



(2018-DoEe)

A/E 25. Find the largest possible positive integer that divides 125, 162 and 259 leaving remainders 5, 6 and 7 respectively. (2016-DXXAAAY, JR711RB)

[*Hint :* HCF of (125 – 5), (162 – 6), (259 – 7)]

A/E 26. Jenny and Sally bought a special 360 days joint membership of a tennis club. Jenny will use the club every alternate day and Sally will use the club every third day. They both use the club on the first day. How many days will neither person use the club in the 360 days ? (2016-JIB3CW8, 1O9W2A0)

[*Hint :* $360 - \frac{360}{2} - \frac{360}{3}$ + HCF of 180 & 120]

Ⓐ 27. Dhudnath has two vessels containing 720 mL and 405 mL of milk respectively. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled.
(2016-RRDXHCG)

[*Hint :* Required number
$= \frac{720}{HCF(720,405)} + \frac{405}{HCF(720,405)}$]

***Note* : Thick Letters (Bold Face) for Standard Questions /** Thin Letters (Normal Face) for Basic Questions

Ⓤ **28. State Fundamental theorem of Arithmetic. Hence, find the number of divisors of 1024.** (2016-27S69RA, F8XQM1K)

Ⓤ **29. An army group of 308 members is to march behind an army band of 24 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march ?**

(2016-ML0NW83; 2015-0OUM5KY)

Ⓐ **30. Write the HCF and LCM of the smallest odd composite number and the smallest odd prime number. If an odd number p divides q^2, then will it divide q^3 also ? Explain.** (2016-VIVMUN; 2015-Y6UPNSO; 2014-3EK66RB)

Ⓐ **31. A sweet shopkeeper prepares 396 gulab jamuns and 342 ras-gullas. He packs them in containers. Each container consists of either gulab jamuns or ras-gullas but have equal number of pieces. Find the number of pieces he should put in each box so that number of boxes are least.**

(2016-LO3AGID; 2015-L2KH55T; 2014-7X8TXXX)

(A/E) **32. What is the HCF and LCM of two prime numbers a and b?**

Ⓐ **33. Three alarm clocks ring at intervals of 6, 9 and 15 minutes respectively. If they start ringing together, after what time will they next ring together?** (2016-7MMS6BF, R4KL3VD; 2015-LA6MRLF, X2MWUF; 2014-VISSRN)

Answers

1. (*c*)	2. (*a*)	3. (*c*)
4. (*d*)	5. *b*	6. 60
7. 55	8. 14	9. 30
10. 23460	11. 90	13. 4608
14. 44	15. 3840	

17. A = 600, B = 270, C = 180

18. 2^8 22. 25 cm

23. Yes, it is possible to be 18 as HCF and 378 as LCM of two numbers.

24. $a = 552$, $b = 276$, $c = 138$, $d = 23$

25. Required number = 12

26. 120 days 27. 25 glasses

28. 9 29. 4

30. HCF = 3, LCM = 9; Yes

31. 18 32. HCF = 1, LCM = ab

33. 90 minutes.

Topic 2. Irrational Number

Revision Notes

A number is called irrational if it cannot be written in the from of $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

$\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc are irrational numbers.

In general, if p is a prime number, than $\sqrt{p}$ is an irrational number.

We can prove $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc. as irrational numbers by the method of contradiction. A theorem is used in this method.

The theorem states as follows :

"Let p be a prime number. If p divides a^2, then p also divides a, where a is a positive integer."

***Example* 1** : Prove that $\sqrt{2}$ is *an* irrational number.

Sol. ***Step* 1** : Let us assume that $\sqrt{2}$ is a rational number such that

$$\sqrt{2} = \frac{r}{s}, \text{ where } r \text{ and } s \text{ are integers and } s \neq 0.$$

Again, suppose that r and s have a common factor other than 1. Then, we divide r and s by the common factor to get $\sqrt{2} = \frac{p}{q}$ where p and q are co-prime.

***Step* 2** : Squaring both the sides, we get

$$\Rightarrow \quad 2 = \frac{p^2}{q^2},$$

$$\Rightarrow \quad 2q^2 = p^2,$$

$$\Rightarrow \quad q^2 = \frac{p^2}{2} \quad \text{... (1)}$$

***Step* 3** : Since q is an integer. So q^2 is also an integer.

Therefore, 2 divides p^2 completely.

(If 2 divides p^2, it also divides p) (Theorem 1.3)

So, 2 is a factor of p ....(A)

That is, we can write $p = 2m$ for some integer m.

Putting $p = 2m$ in (1), we get

$$q^2 = \frac{(2m)^2}{2} = \frac{4m^2}{2} = 2m^2$$

Note : **Thick Letters (Bold Face) for Standard Questions** / Thin Letters (Normal Face) for Basic Questions

$\Rightarrow \qquad m^2 = \frac{q^2}{2}$

Step 4 : Again, m is *an* integer. So, m^2 is also *an* integer.

Therefore, 2 divides q^2 completely.

If 2 dvides q^2, it also divides q (Theorem 1.3)

So, 2 is *a* factor of q ...(B)

Step 5 : From (A) *a*nd (B), we can say that 2 is *a* common factor of p and q. But, this is contrary to the fact that p and q are co-prime.

This contradiction is because of our incorrect assumption that $\sqrt{2}$ is a rational number.

Hence, it is proved that $\sqrt{2}$ is an irrational number.

***Example* 2** : Show that $7-\sqrt{2}$ is irrational.

Sol . Let us assume that $7-\sqrt{2}$ is irrational such that

$7-\sqrt{2} = \frac{p}{q}$, where p and q are co-prime and $q \neq 0$

$\Rightarrow \quad 7 - \frac{p}{q} = \sqrt{2} \quad$ or $\quad \frac{7q-p}{q} = \sqrt{2}$

Since, p and q are integers, we get $\frac{7q-p}{q}$, a rational number. But, it is equal to $\sqrt{2}$. So, $\sqrt{2}$ is also a rational number.

But this contradicts the fact that $\sqrt{2}$ is an irrational number.

This contradiction is because of our incorrect assumption that $7-\sqrt{2}$ is rational.

Hence, it is proved that $7-\sqrt{2}$ is irrational.

EXERCISE 1.2

Competency Based Questions

Multiple Choice Questions

Ⓐ **1. If $p^2 = \frac{32}{50}$, then p is a/an**

(a) whole number (c) rational number
(b) integer (d) irrational number

(CBSE 30-6-1, 2023)

Sol. (c) rational number

Ⓡ **2.** The number $(5 - 3\sqrt{5} + \sqrt{5})$ is :

(a) an integer
(b) a rational number
(c) an irrational number
(d) a whole number ***(CBSE 430-4-1, 2023)***

Sol. (c) an irrational number

Ⓤ **3.** Which of the following is an irrational number ? ***(CBSE Comptt., 2023)***

(a) $\left(2\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2$ (b) $(\sqrt{2}-1)^2$

(c) $\sqrt{2} - (2+\sqrt{2})$ (d) $\frac{(\sqrt{2}+5\sqrt{2})}{\sqrt{2}}$

Sol. (b) $(\sqrt{2}-1)^2$

A/E **4.** Which of the following is NOT an irrational number?

(a) $2 \times \frac{1}{\sqrt{2}}$ (b) $\sqrt{2} \times \frac{1}{2}$

(c) $\sqrt{2} \times \frac{1}{\sqrt{3}}$ (d) $\sqrt{2} \times \frac{1}{\sqrt{2}}$

Sol. (d) $\sqrt{2} \times \frac{1}{\sqrt{2}}$

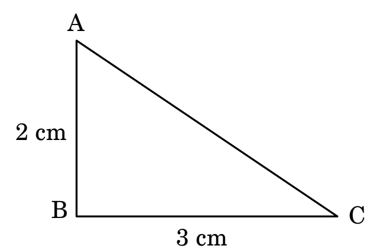
Assertion Reason Type

Following questions consist of two statements — Assertion (A) and Reason (R). Answer these questions, selecting an appropriate option given below :

(a) *Both (A) and (R) are true and (R) is the correct explanation of (A).*
(b) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*
(c) *(A) is true but (R) is false.*
(d) *(R) is true but (A) is false.*

Ⓐ **5. (A) :** The perimeter of ΔABC is a rational number.

(R) : The sum of the squares of two rational numbers is always rational.



(CBSE 30-1-1, 2023)

Ans. (d)

Ⓡ **6. (A) :** 2 is a prime number.

(R) : The square of an irrational number is always a prime number.

Ans. (c)

Ⓤ **7. (A) :** Product of $(2+\sqrt{3})$ and $(3+\sqrt{5})$ is an irrational number.

(R) : Product of two irrational numbers is an irrational number.

Ans. (*c*)

Very Short Answer Type Question

Ⓤ **8.** Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol. $\sqrt{2}$ = 1.4142.... and $\sqrt{3}$ = 1.7320....

Rational number between $\sqrt{2}$ and $\sqrt{3}$ is 1.5.

Short Answer Type Questions - I

Ⓐ **9. Prove that $2 + \sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.** ***(CBSE 30-2-1, 2023)***

Sol. Let us assume that $2 + \sqrt{3}$ is rational.

Let $2 + \sqrt{3} = \dfrac{p}{q}$; $q \neq 0$ and p, q are integers.

$\Rightarrow \quad \sqrt{3} = \dfrac{p - 2q}{q}$

Since p and q are integers, so $p - 2q$ is an integer.

$\Rightarrow \quad \dfrac{p - 2q}{q}$ is a rational number.

$\Rightarrow$ $\sqrt{3}$ is a rational number, which contradicts the given fact that $\sqrt{3}$ is an irrational number.

$\Rightarrow$ $2 + \sqrt{3}$ is an irrational number.

Ⓤ **10. Prove that $6 - \sqrt{7}$ is an irrational number, given that $\sqrt{7}$ is an irrational number.** ***(CBSE 30-2-3, 2023)***

Sol. Let us assume that $6 - \sqrt{7}$ is rational.

$\therefore \quad 6 - \sqrt{7} = \dfrac{p}{q}$; $q \neq 0$ and p, q are integers.

$\Rightarrow \quad \sqrt{7} = \dfrac{6q - p}{q}$

Since p, q are integers, so $6q - p$ is an integer.

$\Rightarrow \quad \dfrac{6q - p}{q}$ is a rational number.

$\Rightarrow$ $\sqrt{7}$ is rational number, which contradicts the given fact that $\sqrt{7}$ is an irrational number.

$\Rightarrow \quad 6 - \sqrt{7}$ is an irrational number

Ⓐ **11. If $\sqrt{2}$ is given as an irrational number, then prove that $(5 - 2\sqrt{2})$ is an irrational number.** ***(CBSE Comptt., 2023)***

Sol. Let us assume that $5 - 2\sqrt{2}$ be a rational number.

$5 - 2\sqrt{2} = \dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

$\Rightarrow \quad \sqrt{2} = \dfrac{5q - p}{2q}$

RHS is a rational number. So, LHS is also a rational number, which contradicts the given fact that $\sqrt{2}$ is an irrational number.

So, our assumption is wrong.

Hence, $5 - 2\sqrt{2}$ is an irrational number.

Short Answer Type Questions - II

Ⓤ **12. Prove that $\sqrt{5}$ is an irrational number. (NCERT TBQ) *(CBSE 30-1-1, 30-4-1, 2023)***

Sol. Let $\sqrt{5}$ be a rational number.

$\therefore \quad \sqrt{5} = \dfrac{p}{q}$, where $q \neq 0$ and let p and q be co-prime.

$5q^2 = p^2 \Rightarrow p^2$ is divisible by $5 \Rightarrow p$ is divisible by 5.

$\Rightarrow \quad p = 5a$, where 'a' is some integer ...(*i*)

$25a^2 = 5q^2 \Rightarrow q^2 = 5a^2 \Rightarrow q^2$ is divisible by $5 \Rightarrow q$ is divisible by 5.

$\Rightarrow \quad q = 5b$, where 'b' is some integer ...(*ii*)

(*i*) and (*ii*) leads to contradiction as 'p' and 'q' are co-prime.

$\therefore \quad \sqrt{5}$ is an irrational number.

Ⓐ **13. Prove that $5 + 6\sqrt{7}$ is irrational.** ***(CBSE Addl. Practice, 2023-24)***

Sol. Let us assume that $5 + 6\sqrt{7}$ is rational.

Let $5 + 6\sqrt{7} = \dfrac{p}{q}$; $q \neq 0$ and p, q are integers.

$\Rightarrow \quad \sqrt{7} = \dfrac{p - 5q}{6q}$

p and q are integers.

$\therefore \quad p - 5q$ is an integer.

$\dfrac{p - 5q}{6q}$ is a rational number.

Note : **Thick Letters (Bold Face) for Standard Questions** / Thin Letters (Normal Face) for Basic Questions

$\Rightarrow$ $\sqrt{7}$ is a rational number, which is a contradiction.

So, our assumption that $5 + 6\sqrt{7}$ is a rational number is wrong.

Hence $5 + 6\sqrt{7}$ is an irrational number.

Ⓡ **14. Prove that $5 - \sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.**

(CBSE 430-1-2, 2023)

Sol. Let us assume that $5 - \sqrt{3}$ is rational number.

$\therefore$ $5 - \sqrt{3} = \frac{p}{q}$; $q \neq 0$ and p, q are integers.

$$\Rightarrow \sqrt{3} = \frac{5q - p}{q}$$

RHS is rational but LHS is irrational.

$\therefore$ Our assumption is wrong.

$\therefore$ $5 - \sqrt{3}$ is an irrational number.

Ⓤ **15. Prove that $4 + 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.**

(CBSE 430-2-1, 2023)

Sol. Let us assume that $4 + 2\sqrt{3}$ is a rational number.

$4 + 2\sqrt{3} = \frac{p}{q}$; $q \neq 0$ and p, q are integers.

$$\Rightarrow \sqrt{3} = \frac{p - 4q}{2q}$$

RHS is rational but LHS is irrational.

$\therefore$ Our assumption is wrong.

Hence, $4 + 2\sqrt{3}$ is an irrational number.

Long Answer Type Question

Ⓐ **16. Prove that $\sqrt{5}$ is an irrational number. Hence, show that $3 + 2\sqrt{5}$ is also an irrational number.**

(2016-2INFC5U, NCERT TBQ)

HINT By contradiction

Sol. Let us suppose that $\sqrt{5}$ is a rational number. Then,

$\sqrt{5} = \frac{p}{q}$, where p and q are co-prime and $q \neq 0$.

Squaring both the sides, we get

$$5 = \frac{p^2}{q^2}$$

$$\Rightarrow 5q^2 = p^2 \qquad \ldots(1)$$

$\Rightarrow$ p^2 is divisible by 5.

So, p is also divisible by 5.

Let $p = 5m$ for some integer m.

Substituting $p = 5m$ in (1), we get

$$5q^2 = (5m)^2$$
$$= 25m^2$$
$$\Rightarrow q^2 = 5m^2$$

$\Rightarrow$ q^2 is divisible by 5.

So, q is also divisible by 5.

Since p and q both are divisible by 5, therefore 5 is a common factor of both p and q. But, this contradicts the assumption that p and q are co-prime.

This is because of our wrong assumption that $\sqrt{5}$ is a rational number.

Hence, $\sqrt{5}$ is an irrational number.

Again, we assume that $3 + 2\sqrt{5}$ is a rational number. Then,

$3 + 2\sqrt{5} = \frac{a}{b}$, where a and b are co-prime and $b \neq 0$.

$$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}.$$

$$\Rightarrow \sqrt{5} = \frac{a - 3b}{2b}.$$

Since a and b are some integers, so $\frac{a - 3b}{2b}$ is a rational number.

But, $\sqrt{5}$ is an irrational number. So, the equality cannot exist.

This is because of our wrong assumption that $3 + 2\sqrt{5}$ is a rational number.

Hence, $3 + 2\sqrt{5}$ is an irrational number.

Practice Exercise

OBJECTIVE TYPE QUESTIONS

I. *Multiple Choice Questions*

Ⓡ 1. $(2+\sqrt{5})(2+\sqrt{5})$ is :

(A) A rational number
(B) A whole number
(C) An irrational number
(D) A natural number

[CBSE-2010-1040117-B1]

Ⓡ 2. $(\sqrt{2}-\sqrt{3})(\sqrt{3}+\sqrt{2})$ is :

(A) A rational number
(B) A whole number
(C) An irrational number
(D) A natural number

[CBSE-2010-1040117-C1]

II. ***Assertion Reason Type***

Following questions consist of two statements — Assertion (A) and Reason (R). Answer these questions, selecting an appropriate option given below :

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*
(*b*) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*
(*c*) *(A) is true but (R) is false.*
(*d*) *(R) is true but (A) is false.*

Ⓤ **3. (A) :** Product of 6 and $\sqrt{7}$ is an irrational number.

(R) : Product of a non-zero rational number and an irrational number is an irrational number.

Ⓡ **4. (A) :** Product of $\frac{1}{2}$ and $\sqrt{5}$ is an irrational number.

(R) : Product of a rational number and an irrational number is an irrational number.

III. ***Very Short Answer Type Questions***

A/E **5.** Prove that $15+17\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

A/E **6.** Prove that $7-3\sqrt{5}$ is an irrational number. It is given that $\sqrt{5}$ is an irrational number.

SHORT ANSWER TYPE QUESTION - I

Ⓐ 7. Check whether the statement below is true or false.

"The square root of every composite number is rational."

Justify your answer by proving rationality or irrationality as applicable. **(*CBSE Addl. Practice, 2023-24*)**

SHORT ANSWER TYPE QUESTION - II

Ⓤ **8.** Prove that $10+2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. **(*CBSE 430-2-3, 2023*)**

Ⓡ 9. **Prove that $\sqrt{3}$ is an irrational number.** **(*CBSE 30-4-2, 30-5-1, 2023*)**

Ⓐ **10.** Prove that $3+7\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number. **(*CBSE 430-1-1, 2023*)**

Answers

1. (C) 2. (A) 3. (*a*) **4.** (*c*)
7. False

Case Study Questions

SUBJECTIVE TYPE QUESTIONS

Ⓐ **1. February 14 is celebrated as International Book Giving Day and many countries in the world celebrate this day. Some people in India also started celebrating this day and donated the following number of books of various subjects to a public library :**

History = 96,
Science = 240,
Mathematics = 336.

These books have to be arranged in minimum number of stacks such that each stack contains books of only one subject and the number of books on each stack is the same.

Based on the above information, answer the following questions :

(*i*) How many books are arranged in each stack ?

Note **: Thick Letters (Bold Face) for Standard Questions /** Thin Letters (Normal Face) for Basic Questions

(*ii*) How many stacks are used to arrange all the Mathematics books ?

(*iii*) (*a*) Determine the total number of stacks that will be used for arranging all the books.

Or

(*iii*) (*b*) If the thickness of each book of History, Science and Mathematics is 1·8 cm, 2·2 cm and 2·5 cm respectively, then find the height of each stack of History, Science and Mathematics books.

(*CBSE Comptt., 2023*)

Sol. (*i*) HCF (96, 240, 336) = 48

So, 48 books are arranged in each stack.

(*ii*) Number of stacks for Mathematics books

$$= \frac{336}{48} = 7$$

(*iii*) (*a*) Total number of stacks

$$= \frac{96}{48} + \frac{240}{48} + \frac{336}{48} = 14$$

Or

(*b*) Height of each stack of History

= 48 × 1.8 = 86.4 cm

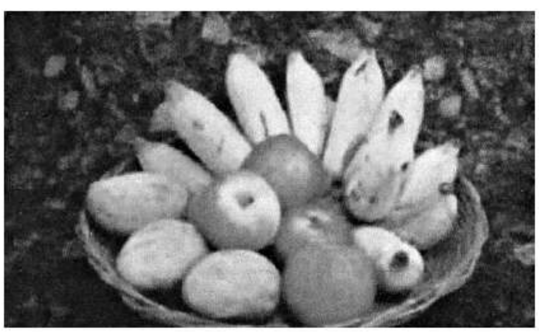
Height of each stack of Science

= 48 × 2.2 = 105.6 cm

Height of each stack of Mathematics

= 48 × 2.5 = 120 cm

Ⓤ **2.** Khushi wants to organize her birthday party. Being health conscious, she decided to serve only fruits in her birthday party. She bought 36 apples and 60 bananas and decided to distribute fruits equally among all.



Based on the above information, answer the following questions :

(*i*) How many guests Khushi can invite at the most ?

(*ii*) How many apples and bananas will each guest get ?

(*iii*) (*a*) If Khushi decides to add 42 mangoes, how many guests Khushi can invite at the most ?

Or

(*b*) If the cost of 1 dozen of bananas is ₹ 60, the cost of 1 apple is ₹ 15 and cost of 1 mango is ₹ 20, find the total amount spent on 60 bananas, 36 apples and 42 mangoes.

(*CBSE 430-4-1, 2023*)

Sol. (*i*) HCF (36, 60) = 12

Khushi can invite at the most 12 guests.

(*ii*) 36 ÷ 12 = 3, 60 ÷ 12 = 5

Each guest will get 3 apples and 5 bananas

(*iii*) (*a*) HCF (36, 60, 42) = 6

Khushi can invite at the most 6 guests

Or

(*b*) Total cost = 5 × 60 + 36 × 15 + 42 × 20

= ₹ 1680

Ⓐ **3.** Three students A, B and C of a school can cycle 48 km, 60 km and 72 km a day respectively around a circular field of circumference 360 km. During the sport day activities of their school, they start cycling together from the same place.

(*a*) After how many rounds they will again meet at the same place?

(*b*) How much distance will be covered by them to meet again?

(*c*) What will be the time taken by 'A' and 'B' to cover the given distance?

Or

(*d*) What will be the time taken by 'C' to cover the given distance ?

Sol. (*a*) 2

(*b*) 720 km

(*c*) 15 days and 12 days

Or

(*d*) 10 days

A/E **4.** Sunita teaches mathematics to some students at her home. She usually uses playway activities to teach the concepts. One day, she planned a game for prime numbers. She first announce the number 2 in the class and asked the first student to multiply it by any prime number and pass to the second student. Second student again multiplied the product with a prime number and passed it on the third student and the process continued. After multiplying the product by a prime number, the last student obtained 103950. Teacher announced this number in the class and asked some questions as given below :

(*a*) What is the least prime number used by students?

(*b*) Which of the prime numbers have been used minimum number of times?

(*c*) Which prime number has been used maximum number of times? How many students are there in the class?

Sol. (*a*) 3, (*b*) 7 and 11, (*c*) 3; 7

Ⓐ **5.** A certain parade has been planned to be organized in the form of following two groups:

(1) Group A having 624 soldiers behind a band of 32 numbers.

(2) Group B having 468 soldiers behind 228 bikers. These two groups are to march in the same number of columns.

(*a*) What is the maximum number of columns in which members of Group A can march?

(*b*) What is the maximum number of columns in which members of Group B can march?

(*c*) What should be subtracted from the number of Group B soldiers and number of bikers so that their maximum number of columns is equal to the maximum number of columns of Group A?

Or

(*d*) What should be added to the number of Group B soldiers and number of bikers so that their maximum numbers of columns is equal to the maximum number of columns of Group A?

Sol. (*a*) 16, (*b*) 2, (*c*) 4 soldiers and 2 bikers, *Or* (*d*) 12 soldiers and 12 bikers.

6. Some students have donated 96 English books, 240 Hindi books and 336 Mathematics books to the School Bank. These books are to be stacked in such a way that number of copies in each stack is the same and in one stock only books of one subject are there.

(*i*) To keep the number of stacks minimum, in what way the books be kept in each stack?

(*ii*) What is the number of books in each stack in such a case?

(*iii*) What is the minimum number of stacks?

Sol. (*i*) To keep the number of stack minimum, maximum number of books should be kept in each stack.

(*ii*) Maximum number of books in each stack is the HCF of 96, 240 and 336. As $96 = 48 \times 2$, $240 = 48 \times 5$ and $336 = 48 \times 7$, so HCF $= 48$.

(*iii*) Minimum number of stacks

$$= \frac{96}{48} + \frac{240}{48} + \frac{336}{48}$$

$$= 2 + 5 + 7 = 14.$$

A/E **7. As an adventure, three friends Abdul, Naresh and John start cycling around a circular field of circumference 360 km at the same time and from the same place. They can cycle around the field at the rate of 48 km, 60 km and 72 km a day respectively.**

(*i*) After covering how much distance, they will meet again at the same place?
(*ii*) For how many days each of them must have cycled by that time?
(*iii*) How many rounds of the field, these three must have completed in this process?

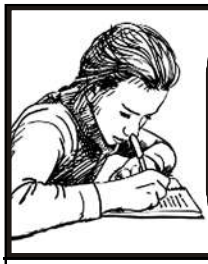
Sol. (*i*) Required distance = LCM of 48, 60 and 72. As $48 = 2^4 \times 3$, $60 = 2^2 \times 3 \times 5$ and $72 = 2^3 \times 3^2$, so LCM $= 2^4 \times 3^2 \times 5 = 720$. Thus required distance is 720 km.

(*ii*) Number of days for Abdul $= \dfrac{720}{48} = 15$.

Number of days for Naresh $= \dfrac{720}{60} = 12$.

Number of days for John $= \dfrac{720}{72} = 10$.

(*iii*) Number of rounds $= \dfrac{720}{360} = 2$.



Chapter TEST

STANDARD

Time allowed : 1½ hours *Maximum Marks :* 30

General instructions :

(*i*) Do all the questions given in the chapter test.
(*ii*) Section–A : Questions 1 to 7 is of 1 mark each.
(*iii*) Section–B : Questions 8 and 9 is of 2 marks each.
(*iv*) Section–C : Questions 10 to 12 is of 3 marks each.
(*v*) Section–D : Questions 13 and 14 is of 5 marks each.

SECTION–A

Multiple Choice Questions

Ⓡ 1. (HCF × LCM) for the numbers 70 and 40 is : ***(CBSE 430-4-1, 2023)***
(*a*) 10 (*b*) 280 (*c*) 2800 (*d*) 70

Ⓐ 2. HCF of $(3^4 \times 2^2 \times 7^3)$ and $(3^2 \times 5 \times 7)$ is : ***(CBSE 430-5-3, 2023)***
(*a*) 630 (*b*) 63 (*c*) 729 (*d*) 567

Ⓡ 3. LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is : ***(CBSE 430-5-1, 2023)***
(*a*) 40 (*b*) 560 (*c*) 1680 (*d*) 1120

Ⓡ 4. If n is a natural number, then 8^n cannot end with digit ***(CBSE 430-6-1, 2023)***
(*a*) 0 (*b*) 2 (*c*) 4 (*d*) 6

Ⓡ 5. If HCF (72, 120) = 24, then LCM (72, 120) is : ***(CBSE 430-6-1, 2023)***
(*a*) 72 (*b*) 120 (*c*) 360 (*d*) 9640

Ⓡ 6. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, the other number is : ***(CBSE Comptt., 2023)***
(*a*) 36 (*b*) 45 (*c*) 9 (*d*) 81

Assertion Reason Type

The following question consists of two statements — Assertion (A) and Reason (R) Answer the questions, selecting an appropriate option given below :

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*
(*b*) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*
(*c*) *(A) is true but (R) is false.*
(*d*) *(R) is true but (A) is false.*

Ⓤ 7. **(A)** : HCF of any two prime numbers is 1.

(R) : HCF of two coprime numbers is 1.

SECTION–B

Ⓤ **8.** Find the HCF and LCM of 72 and 120. ***(CBSE 30-6-1, 2023)***

Ⓤ **9.** Give example of two irrational numbers whose
(*i*) sum is a rational number.
(*ii*) product is an irrational number. ***(CBSE Comptt., 2023)***

SECTION–C

Ⓤ **10.** Prove that $2 - 3\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number. ***(CBSE 430-2-2, 2023)***

Ⓤ **11.** Prove that $(7 - 2\sqrt{3})$ is an irrational number, given that $\sqrt{3}$ is an irrational number. ***(CBSE 430-5-1, 2023)***

Ⓐ **12.** Prove that $7 + 4\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number. ***(CBSE 430-6-3, 2023)***

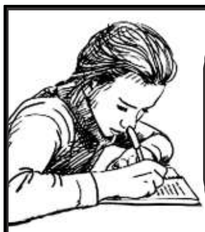
SECTION–D

Ⓐ **13.** Find the HCF of 256 and 36. Also, find their LCM and verify that HCF × LCM = Product of the two numbers.

Ⓐ **14.** Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7 + 2\sqrt{3}$ is also an irrational number.

Answers

1. (*c*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*c*)
6. (*c*)	**7.** (*a*)	**8.** 24, 360	**13.** HCF = 4, LCM = 2304	



Chapter TEST
BASIC

Time allowed : 1½ hours *Marks :* 24

General instructions :

(*i*) Do all the questions given in the chapter test.
(*ii*) Section–A : Questions 1 to 8 is of 1 mark each.
(*iii*) Section–B : Question 9 is of 2 marks each.
(*iv*) Section–C : Questions 10 to 12 is of 3 marks each.
(*v*) Section–D : Question 13 is of 5 marks each.

SECTION–A

Multiple Choice Questions

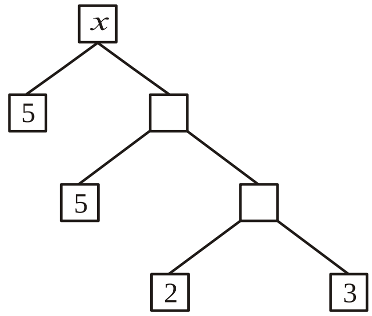
Ⓤ **1.** (HCF × LCM) for the numbers 30 and 70 is : ***(CBSE 430-4-2, 2023)***
(*a*) 2100 (*b*) 21 (*c*) 210 (*d*) 70

Ⓤ **2.** The HCF of the smallest 2-digit number and the smallest composite number is
(*a*) 4 (*b*) 20 (*c*) 2 (*d*) 10
(CBSE 430-6-2, 2023)

Ⓤ **3.** The prime factorisation of the number 2304 is : ***(CBSE 430-6-1, 2023)***
(*a*) $2^8 \times 3^2$ (*b*) $2^7 \times 3^3$ (*c*) $2^8 \times 3^1$ (*d*) $2^7 \times 3^2$

Ⓤ **4.** The prime factorisation of the number 5488 is ***(CBSE 430-6-3, 2023)***
(*a*) $2^3 \times 7^3$ (*b*) $2^4 \times 7^3$ (*c*) $2^4 \times 7^4$ (*d*) $2^3 \times 7^4$

Ⓤ 5. The value of x in the factor tree is :



(*a*) 30

(*b*) 150

(*c*) 100

(*d*) 50

Ⓤ 6. A pair of irrational numbers whose product is a rational number is :

(*a*) $\sqrt{16}$, $\sqrt{4}$ (*b*) $\sqrt{5}$, $\sqrt{2}$ (*c*) $\sqrt{3}$, $\sqrt{27}$ (*d*) $\sqrt{36}$, $\sqrt{2}$

Assertion Reason Type

Following questions consist of two statements — Assertion (A) and Reason (R). Answer these questions, selecting an appropriate option given below :

(*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

(*b*) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*

(*c*) *(A) is true but (R) is false.*

(*d*) *(R) is true but (A) is false.*

A/E 7. **(A)** : HCF of 27 and 81 is 27.

(R) : 9 exactly divides both 27 and 81.

Very Short Answer Type Question

Ⓐ 8. What is the HCF of p and q if $q = 11p$?

SECTION–B

Ⓤ 9. Find the HCF and LCM of 72 and 120. ***(CBSE 30-6-1, 2023)***

SECTION–C

Ⓤ 10. Prove that $\frac{7\sqrt{3}}{8}$ is an irrational number.

Ⓐ 11. Prove that $\sqrt{2}$ is an irrational number. ***(CBSE 30-6-1, 2023)***

Ⓤ 12. Prove that $2 + 3\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number. ***(CBSE Comptt., 2023)***

SECTION–D

Ⓐ 13. Prove that $\sqrt{5}$ is an irrational number. Hence, show that $5+4\sqrt{5}$ is also an irrational number.

Answers

1. (*a*)	2. (*c*)	3. (*a*)	4. (*b*)	5. (*b*)	6. (*c*)
7. (*b*)	8. p	9. HCF = 24, LCM = 360			

Practice Exercise 1.1

1. (*c*) 2800

Explanation : We can use the relationship between LCM and HCF, which states that LCM $(a, b) \times$ HCF $(a, b) = a \times b$.

We are given the numbers 70 and 40. We can find their HCF and LCM as follows :

$70 = 2 \times 5 \times 7$

$40 = 2^3 \times 5$

HCF $(70, 40) = 2 \times 5 = 10$

LCM $(70, 40) = 2^3 \times 5 \times 7 = 280$

Now, we can find the product of HCF and LCM as follows :

HCF $(70, 40) \times$ LCM $(70, 40) = 10 \times 280 = 2800$

2. (*a*) 2100

Explanation : We can find the HCF and LCM of 30 and 70 using the prime factorisation method as follows :

$30 = 2 \times 3 \times 5$

$70 = 2 \times 5 \times 7$

HCF $(30, 70) = 2 \times 5 = 10$

LCM $(30, 70) = 2 \times 3 \times 5 \times 7 = 210$

Therefore, (HCF × LCM) for the numbers 30 and 70 is :

$10 \times 210 = 2100$

3. (*c*) 1680

Explanation : We can find the LCM of $(23 \times 3 \times 5)$ and $(24 \times 5 \times 7)$ using the prime factorisation method as follows :

$23 \times 3 \times 5 = 2 \times 3 \times 5 \times 23$

$24 \times 5 \times 7 = 2^3 \times 3 \times 5 \times 7$

To find the LCM, we take the highest power of each prime factor involved in the numbers :

LCM $(23 \times 3 \times 5, 24 \times 5 \times 7) = 2^3 \times 3 \times 5 \times 7 \times 23$

Therefore, the LCM of $(2^3 \times 3 \times 5)$ and $(24 \times 5 \times 7)$ is $2^3 \times 3 \times 5 \times 7 \times 23 = 1680$.

4. (*d*) *(R) is true but (A) is false.*

7. LCM of 12, 16, 24 = 48.

Required number is 48 + 7 = 55.

8. We have to find HCF of $85 - 1 = 84$ and $72 - 2 = 70$.

HCF of 84 and 70 = 14

9. Let the numbers be $2x$, $3x$.

LCM $= 6x = 180 \Rightarrow x = 30$

$\therefore$ Numbers are 60, 90

HCF (60, 90) = 30

12. Take a number which is not a perfect square but is a composite number. For example, 6.

Assume $\sqrt{6} = \frac{a}{b}$, where $b \neq 0$, a and b are co-prime.

Write $b\sqrt{6} = a$ and square on both sides to get $6b^2 = a^2$.

Write that as a^2 is divisible by 2 and 3 which are both prime numbers, a is also divisible by both 2 and 3. Hence conclude that a is divisible by 6.

Write $a = 6c$, where c is an integer and squares on both sides to get $a^2 = 36c^2$.

Replaces a^2 with $6b^2$ from step 2 to get $6b^2 = 36c^2$ and solve it to get $b^2 = 6c^2$.

Write that as b^2 is divisible by 2 and 3 which are both prime numbers, b is also divisible by both 2 and 3. Hence conclude that b is divisible by 6.

Write that 2 and 3 divide both a and b which contradicts the assumption that a and b are co-prime and hence $\sqrt{6}$ is irrational. Conclude that the given statement is false.

10.
$$92 = 2 \times 2 \times 23$$
$$510 = 2 \times 3 \times 5 \times 17$$
$$\text{HCF} = 2$$
$$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$$

11.
$$540 = 2^2 \times 3^3 \times 5$$
$$630 = 2 \times 3^2 \times 5 \times 7$$
$$\text{HCF} = 2 \times 3^2 \times 5 = 90$$

12. $15^n = (3 \times 5)^n = 3^n \times 5^n$

13.
$$576 = 2^6 \times 3^2$$
$$512 = 2^9$$
$$\text{LCM} = 512 \times 9 = 4608$$

14.
$$660 = 2^3 \times 3 \times 5 \times 11$$
$$704 = 2^3 \times 11$$
$$\text{HCF } (660, 704) = 2^2 \times 11 = 44$$

15.
$$580 = 2^5 \times 3 \times 5$$
$$256 = 2^8$$
$$\text{HCF } (480, 256) = 2^8 \times 3 \times 5 = 3840$$

Note **: Thick Letters (Bold Face) for Standard Questions /** Thin Letters (Normal Face) for Basic Questions

17. Hint : Find the HCF and LCM of A, B and C from the prime factorisation as :

HCF = $2^p \times 3^p \times 5^p$

LCM = $2^r \times 3^r \times 5^q$

From the given information, infer that HCF of A, B and C is 30 and equate it to the HCF obtained in step 1 to get the value of p as :

$$2^p \times 3^p \times 5^p = 30$$

$$\Rightarrow \quad (2 \times 3 \times 5)^p = (2 \times 3 \times 5)^1$$

$$\Rightarrow \quad p = 1$$

From the given information, infer that LCM of A, B and C is 5402 – 2 = 5400.

Equate it to the LCM obtained in step 1 to get the values of q and r as :

$$2^r \times 3^r \times 5^q = 5400$$

$$\Rightarrow (2 \times 3)^r \times (5)^q = (2 \times 3)^3 \times (5)^2$$

$$\Rightarrow \quad q = 2 \text{ and } r = 3$$

Substitutes the values of p, q and r to find the values of A, B and C as :

$$A = 2^3 \times 3^1 \times 5^2 = 600$$
$$B = 2^1 \times 3^3 \times 5^1 = 270$$
$$C = 2^2 \times 3^2 \times 5^1 = 180$$

18. Find the prime factorisation of 12^4 as $(2^8 \times 3^4)$.

Find the prime factorisation of 6^4 as $(2^4 \times 3^4)$ and the prime factorisation of 8^2 as 2^6.

Compare the prime factorisations of 6^4, 8^2 and 12^4 and identify that 256 or equivalently, 2^8 is the smallest value of k.

Practice Exercise 1.2

1. (*c*) *(A) is true but (R) is false.*

8. Let us assume that $10 + 2\sqrt{3}$ is a rational number.

$10 + 2\sqrt{3} = \dfrac{p}{q}$; $q \neq 0$ and p, q are integers.

$$\Rightarrow \quad \sqrt{3} = \frac{p - 10q}{2q}$$

RHS is rational but LHS is irrational.

$\therefore$ Our assumption is wrong.

Hence $10 + 2\sqrt{3}$ is an irrational number.

9. Let $\sqrt{3}$ be a rational number.

$\therefore \quad \sqrt{3} = \dfrac{p}{q}$, where $q \neq 0$ and let p and q be co-prime.

$3q^2 = p^2 \Rightarrow p^2$ is divisible by 3 $\Rightarrow p$ is divisible by 3.

$\Rightarrow \quad p = 3a$, where 'a' is some integer.

$9a^2 = 3q^2 \Rightarrow q^2 = 3a^2 \Rightarrow q^2$ is divisible by 3 $\Rightarrow q$ is divisible by 3.

$\Rightarrow \quad q = 3b$, where 'b' is some integer.

(*i*) and (*ii*) leads to contradiction as 'p' and 'q' are co-primes.

$\therefore \quad \sqrt{3}$ is an irrational number.

10. Let us assume that $3 + 7\sqrt{2}$ is a rational number.

$\Rightarrow 3 + 7\sqrt{2} = \dfrac{p}{q}$, p, q are integers and $q \neq 0$

$$\Rightarrow \sqrt{2} = \frac{p - 3q}{7q}$$

RHS is rational but LHS is irrational.

$\therefore$ Our assumption is wrong.

Hence, $3 + 7\sqrt{2}$ is an irrational number.

Chapter Test (Standard)

1. (*c*) 2800
2. (*b*) 63
3. (*c*) 1680
4. (*a*) 0
5. (*c*) 4
6. (*c*) 9
7. (*a*) *Both (A) and (R) are true and (R) is the correct explanation of (A).*

8.
$$72 = 2^3 \times 3^2$$
$$120 = 2^3 \times 3 \times 5$$
$$\text{HCF} = 24$$
$$\text{LCM} = 360$$

9. (*i*) Any relevant example.
   (*ii*) Any relevant example.

10. Let us assume that $2 - 3\sqrt{5}$ is a rational number.

$2 - 3\sqrt{5} = \frac{p}{q}$; $q \neq 0$ and p, q are integers.

$\Rightarrow \quad \sqrt{5} = \frac{2q - p}{3q}$

RHS is rational but LHS is irrational.

$\therefore$ Our assumption is wrong.

Hence $2 - 3\sqrt{5}$ is an irrational number.

11. Let us assume that $7 - 2\sqrt{3}$ is a rational number.

$\Rightarrow \quad 7 - 2\sqrt{3} = \frac{a}{b}$, where a and b are integers, $b \neq 0$

$\Rightarrow \quad \sqrt{3} = \frac{7b - a}{2b}$.

RHS is a rational number but LHS is irrational.

$\therefore$ Our assumption is wrong.

Hence, $7 - 2\sqrt{3}$ is irrational.

12. Let us assume that $7 + 4\sqrt{5}$ is a rational number.

$\Rightarrow \quad 7 + 4\sqrt{5} = \frac{p}{q}$, $q \neq 0$ and p, q are integers.

$\Rightarrow \quad \sqrt{5} = \frac{p - 7q}{4q}$.

Clearly, $\frac{p - 7q}{4q}$ is rational but $\sqrt{5}$ is irrational.

Our assumption is wrong.

$\Rightarrow \quad 7 + 4\sqrt{5}$ is irrational.

13. HCF = 4, LCM = 2304

Chapter Test (Basic)

1. (*a*) 2100 2. (*c*) 2 3. (*a*) $2^8 \times 3^2$
4. (*b*) $2^4 \times 7^3$ 5. (*b*) 150
6. (*c*) $\sqrt{3}, \sqrt{27}$
7. (*b*) *Both (A) and (R) are true but (R) is not the correct explanation of (A).*
8. *p*
9. HCF = 24, LCM = 360

11. Let $\sqrt{2}$ be a rational number.

$\therefore \quad \sqrt{2} = \frac{p}{q}$ where $q \neq 0$ and let p and q be co-prime.

$2q^2 = p^2 \Rightarrow p^2$ is divisible by 2 $\Rightarrow p$ is divisible by 2.

$\Rightarrow \quad p = 2a$, where 'a' is some integer.

$4a^2 = 2q^2 \Rightarrow q^2 = 2a^2 \Rightarrow q^2$ is divisible by 2 $\Rightarrow q$ is divisible by 2.

$\Rightarrow \quad q = 2b$, where 'b' is some integer.

(*i*) and (*ii*) leads to contradiction as 'p' and 'q' are co-prime.

$\therefore \quad \sqrt{2}$ is an irrational number.

12. Let us assume that $2 + 3\sqrt{3}$ is a rational number.

$\Rightarrow \quad 2 + 3\sqrt{3} = \frac{p}{q}$; p and q are integers and $q \neq 0$

$\Rightarrow \quad \sqrt{3} = \frac{p - 2q}{3q}$

RHS is a rational but LHS is irrational.

$\therefore$ Our assumption is wrong.

Hence, $2 + 3\sqrt{3}$ is an irrational number.