C.P. Digest

# Mathematics

(Standard)

## Class-X

## Sample Question Paper 4

(Answers)

**1.** (B) Composite number

**2.** (A) First choice : $\pi$; Second choice : 20+16; Third choice : 50-1; Correct Answer : 49

**3.** (D) $-2 < p < 2$

**4.** (A) $4x - x + 3y - 3y = 41 - 26$

**5.** (C) 12

**6.** (A) Triangles are similar by SAS.

**7.** (D) For triangles to be similar, the measure of $\angle A = 100°$.

**8.** (A) $m = 0.5 + n$

**9.** (A) Statement I and II

**10.** (C) $\dfrac{a}{\sqrt{b^2 - a^2}}$

**11.** (C) sec 90°

**12.** (A) $\dfrac{1}{\sqrt{2}}$

**13.** (B) 1232 cm$^2$

**14.** (C) 3 : 1

**15.** (A) 8 : 27

**16.** (B) Median because the data has extreme data points.

**17.** (D) The central tendency of the data increases by 0.2 as the median increases by 0.2.

**18.** (B) Blue

**19.** (C) (A) is true but (R) is false.

**20.** (C) (A) is true but (R) is false.

**21.** Given system of linear equations is $2x + 3y = 1$ and $(k - 1)\,x + (2k + 1)\,y = (k - 1)$. This, system has no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{2}{k-1} = \frac{3}{(2k+1)} \neq \frac{1}{k-1}$$

or $3k - 3 = 4k + 2$

or $4k - 3k = -3 - 2$

$\Rightarrow$ $k = -5$.

Now, for $k = -5$,

$$\frac{a_1}{a_2} = \frac{2}{-5-1} = \frac{1}{-3},\ \frac{b_1}{b_2} = \frac{3}{2\times(-5)+1} = \frac{-1}{3} \text{ and } \frac{c_1}{c_2} = \frac{1}{-5-1} = \frac{-1}{6}.$$

Thus, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$.

Hence, $k = -5$, is the answer.

**22.** ***Hint :*** $\Delta AEF \sim \Delta DEC$ shall be shown by using AAA similarity and then the result will be obtained.

In $\Delta AEF$ and $\Delta DEC$, we have :

$\angle A = \angle D$ (Given)

and $\angle AEF = \angle DEC$ (Vertically opposite angles)

So, $\Delta AEF \sim \Delta DEC$ (By AA similarity criterion)

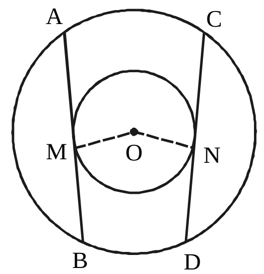
Therefore, $\dfrac{AE}{DE} = \dfrac{AF}{DC}$

$\Rightarrow$ $AE \times DC = DE \times AF$, **proved.**

***Or***

First four terms are 1.25, 1.25 – 0.25, 1.25 – 0.25 – 0.25 and 1.25 – 0.25 – 0.25 – 0.25, *i.e.*, 1.25, 1.00, 0.75 and 0.50.

**23.** ***Hint :*** All such chords will be at a distance which is equal to radius of the smaller circle and hence will be equal.




Join OM and ON.

$$OM = ON \qquad \text{(Radii of a circle)}$$

That is, AB and CD are equidistant from the centre.

So, $AB = CD.$

Thus, all such chords will be equal, proved.

**24.** We have :

$$\sin(A + B) = 1.$$

$\Rightarrow$ $\sin(A + B) = \sin 90° \qquad [\because 1 = \sin 90°]$

$\Rightarrow$ $A + B = 90°$ ...(1)

Also, $\tan(A - B) = \dfrac{1}{\sqrt{3}}.$

$\Rightarrow$ $\tan(A - B) = \tan 30° \qquad [\because \dfrac{1}{\sqrt{3}} = \tan 30°]$

$\Rightarrow$ $A - B = 30°$ ...(2)

Adding (1) and (2), we get

$$A + B = 90°$$
$$A - B = 30°$$
$$2A = 120°$$

$\Rightarrow$ $A = \dfrac{120°}{2} = 60°.$

Putting A = 60° in equation (1), we get

$$60° + B = 90°$$

$\Rightarrow$ $B = 90° - 60° = 30°.$

Hence, A = 60° and B = 30°.

**25.** Volume of a lead ball $= \dfrac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 \text{ cm}^3.$$

Volume of the rectangular solid $= 88 \times 42 \times 21 \text{ cm}^3.$

So, number of lead balls $= \dfrac{\text{Volume of rectangular solid}}{\text{Volume of a ball}}.$

$$= \frac{88 \times 42 \times 21 \times 3 \times 7}{4 \times 22 \times 2.1 \times 2.1 \times 2.1}.$$

$$= \frac{42 \times 21 \times 21 \times 10 \times 10 \times 10}{21 \times 21 \times 21} = 2000.$$

***Or***

Let the radius of the hemisphere be $r$.

Then, volume $= \dfrac{2}{3}\pi r^3$

and area $= 2\pi r^2 + \pi r^2 = 3\pi r^2$

But, volume = Area

So, $\frac{2}{3}\pi r^3 = 3\pi r^2$

$\Rightarrow$ $\frac{r^3}{r^2} = \frac{3\pi \times 3}{2\pi}$

$\Rightarrow$ $r = \frac{9}{2}$

Hence, diameter of the required hemisphere is 9 units.

**26.** ***Hint :*** LCM of 8 and 12 will be divided by 8 and 12 and then results will be added.

Required number of equal pens and notepads for least number of packs of each type is the LCM of 8 and 12.

Now, $8 = 2 \times 2 \times 2 = 2^3$

and $12 = 2 \times 2 \times 3 = 2^2 \times 3$

So, $\text{LCM} = 2^3 \times 3 = 24.$

Hence, least number of packs of pens $= \frac{24}{8} = 3$

and least number of packs of notepads $= \frac{24}{12} = 2.$

**27.** $\alpha$ and $\beta$ are zeroes of $4x^2 + 12x + 9$.

So, $\alpha + \beta = -\frac{12}{4} = -3$

and $\alpha\beta = \frac{9}{4}.$

Now, sum of $(\alpha - 1)$ and $(\beta - 1)$ $= \alpha - 1 + \beta - 1$
$= \alpha + \beta - 2$
$= -3 - 2 = -5$

and product of $(\alpha - 1)$ and $(\beta - 1)$ $= (\alpha - 1)(\beta - 1)$
$= \alpha\beta - \alpha - \beta + 1$
$= \alpha\beta - (\alpha + \beta) + 1$
$= \frac{9}{4} - (-3) + 1$
$= \frac{9}{4} + 3 + 1$
$= \frac{9}{4} + 4 = \frac{25}{4}.$

So, required polynomial is $x^2 - (-5)\,x + \frac{25}{4} = x^2 + 5x + \frac{25}{4}$,

which can also be represented as $4x^2 + 20x + 25$.

**28.** Given pair of linear equations :

$2x + y = 8$ or $2x + y - 8 = 0$

and $5x - 2y = c$ or $5x - 2y - c = 0$

We have : $a_1 = 2,\ b_1 = 1,\ c_1 = -8$

and $a_2 = 5,\ b_2 = -2,\ c_2 = c.$

Condition for unique solution

Here, $\frac{2}{5} \neq \frac{1}{-2}$ *i.e.,* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ for all values of $c$, so this pair of linear equations will have a unique solution for every real value of $c$.

***Or***

Given pair of linear equations is

$$2x + 3y = 7$$

and $$2\alpha x + (\alpha + \beta)y = 28$$

has infinite number of solutions.

So, $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{2\alpha} = \frac{3}{\alpha + \beta} = \frac{7}{28}$$

From $\frac{2}{2\alpha} = \frac{7}{28}$, we get $\frac{1}{\alpha} = \frac{1}{4} \Rightarrow \alpha = 4$ and from $\frac{3}{\alpha+\beta} = \frac{7}{28}$, we get

$$\alpha + \beta = 12 \qquad \text{...(1)}$$

Putting $\alpha = 4$ in (1), we get

$$\beta = 12 - 4 = 8.$$

So, $\alpha = 4$ and $\beta = 8$ are required values.

**29.** We have :

$\angle ORD = \angle OSD = 90°$ (Angles between radii and tangents)

Also, $\angle RDS = 90°$. (Given)

So, $\angle ROS = 360° - 90° - 90° - 90° = 90°$ ...(1)

Also, DS = DR and OR = OS ...(2)

So, OSDR is a square. [From (1) and (2)]

AS = AP. (Tangents from an external point are equal)

BP = QB = 25 cm.

(Tangents from an external point are equal)

$\therefore$ QC = CR = (38 – 25) cm = 13 cm.

(Tangents from an external point are equal)

So, RD = DS = (28 – 13) cm = 15 cm.

(Tangents from an external point are equal)

So, radius of the circle = OS = RD = 15 cm. (OSDR is a square)

**30.** $$\cos (A + B) = \frac{1}{2}.$$

So, $A + B = 60°$ ...(1)

Also, $$\sin (A - B) = \frac{1}{2}.$$

So, $A - B = 30°$ ...(2)

Adding eqns. (1) and (2), we get

$$2A = 90° \Rightarrow A = \frac{90°}{2} = 45°.$$

Putting A = 45° in eqn. (1), we get

$$45° + B = 60°$$

$$\Rightarrow B = 60° - 45° = 15°.$$

***Or***

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{2+3}{6}}{\frac{6-1}{6}}$$

$$= \frac{5}{6} \times \frac{6}{5} = 1$$

$$= \tan 45° \qquad [\because \tan 45° = 1]$$

$$\Rightarrow \quad A + B = 45°.$$

**31.** In a leap year there are 29 days in the month of February.

If February starts with Monday, the last day will be Monday and hence there will be 5 Mondays. So,

(*a*) P(Its last is Monday) = 1.

(*b*) P(It has 5 Mondays) = 1.

(*c*) P(It has 5 Sundays) = 0.

**32.** Let monthly fixed amount be ₹ $x$ and that of each extra minute be ₹ $y$.

So, as per given conditions, we get :

$$x + (370 - 100)y = 433$$

$$\Rightarrow \quad x + 270y = 433 \qquad ...(1)$$

and $$x + (300 - 100)y = 398$$

$$\Rightarrow \quad x + 200y = 398 \qquad ...(2)$$

Subtracting (2) from (1), we get

$$70y = 35 \quad \Rightarrow \quad y = \frac{35}{70} = \frac{1}{2}.$$

Putting $y = \frac{1}{2}$ in (1), we get

$$x + 270 \times \frac{1}{2} = 433$$

$$\Rightarrow \quad x + 135 = 433$$

$$\Rightarrow \quad x = 433 - 135 = 298.$$

So, amount to be paid by Kareena for

$$400 \text{ minutes} = 298 + (400 - 100) \times \frac{1}{2}$$

$$= 298 + \frac{300}{2}$$

$$= 298 + 150 = 448.$$

Thus, amount to be paid is ₹ 448.

***Or***

Let the length and breadth of the rectangle be $x$ m and $y$ m respectively. Then, we get :

$$\text{Area of rectangle} = xy \text{ m}^2.$$

By Ist condition,

$$xy - (x + 5)(y - 4) = 160$$

$$\Rightarrow \quad xy - xy + 4x - 5y + 20 = 160$$

$$\Rightarrow \quad 4x - 5y = 140 \qquad ...(1)$$

By IInd condition,

$$xy - (x - 10)(y + 2) = 100$$

$$\Rightarrow \quad xy - xy - 2x + 10y + 20 = 100$$

$$\Rightarrow \quad -2x + 10y = 80$$

$$\Rightarrow \quad 2x - 10y = -80$$

$$\Rightarrow \quad x - 5y = -40 \quad \text{...(2)}$$

Subtracting (2) from (1), we get

$$3x = 180$$

$$\Rightarrow \quad x = \frac{180}{3} = 60.$$

Putting $x = 60$ in (1), we get

$$4 \times 60 - 5y = 140$$

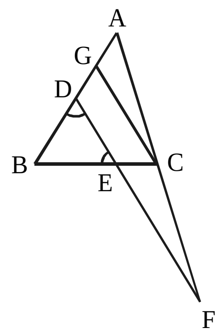
$$\Rightarrow \quad -5y = 140 - 240$$

$$\Rightarrow \quad -5y = -100$$

$$\Rightarrow \quad y = \frac{-100}{-5} = 20.$$

Thus, length is 60 m and breadth is 20 m.

**33.** **Construction :** On AB, take a point G such that CG || DF.




In $\Delta$BDE, $\angle E = \angle D$ (Given) ...(1)

So, BD = BE ...(2)

From $\Delta$BCG, we have :

DE || GC.

So, $\dfrac{BE}{EC} = \dfrac{BD}{DG}$

But BD = BE [From (2)]

So, EC = DG.

$\Rightarrow$ BE = DG (E is mid-point of BC) ...(3)

Now, CG || FD (By construction)

So, $\Delta ACG \sim \Delta AFD$.

$$\Rightarrow \quad \frac{AC}{AF} = \frac{AG}{AD}$$

So, $$1 - \frac{AC}{AF} = 1 - \frac{AG}{AD}$$

$$\Rightarrow \quad \frac{AF - AC}{AF} = \frac{AD - AG}{AD}.$$

$$\Rightarrow \quad \frac{CF}{AF} = \frac{DG}{AD}$$

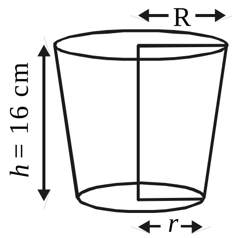
$$\Rightarrow \quad \frac{AF}{CF} = \frac{AD}{DG}$$

$$\Rightarrow \quad \frac{AF}{CF} = \frac{AD}{BE}. \quad \text{[From (3)], } \textbf{proved.}$$

**34.** Here, $2\pi r = 33$

So, $$2 \times \frac{22}{7} \times r = 33$$

or $$r = \frac{\overset{3}{\cancel{33}} \times 7}{2 \times \underset{2}{\cancel{22}}}$$




or $r = 5.25$ cm

Also, $2\pi R = 44$

or $2 \times \frac{22}{7} \times R = 44$

or $R = 7$ cm

Also, $h = 16$ cm (Given)

So, $\text{Volume} = \frac{\pi h}{3}[r^2 + R^2 + Rr]$

$= \frac{22}{7} \times \frac{1}{3} \times 16[(5.25)^2 + 7^2 + 7(5.25)]$ cm$^3$

$= 8625$ cm$^3$

Now, slant height $l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{16^2 + (7 - 5.25)^2} = \sqrt{256 + (1.75)^2}$

$= \sqrt{256 + 3.0625} = \sqrt{258 + .0625}$

$= 16.06 = 16.1$ cm (approx.)

Surface area $= \pi r^2 + \pi R^2 + \pi l(R + r)$

$= \frac{22}{7} \times (5.25)^2 + \frac{22}{7}(7)^2 + \frac{22}{7} \times 16.1 \times (7 - 5.25)$

$= 86.54 + 154 + \frac{22}{7} \times 16.1 \times 12.25$

$= 860.14$ cm$^2$ (approx.)

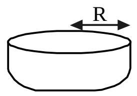
***Or***

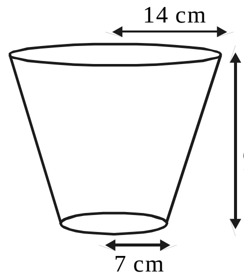
For bucket $h = 16$ cm,

$r_1 = 14$ cm

and $r_2 = 7$ cm

$V_1 = \text{Volume of bucket} = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$




Let R cm be the radius of hemispherical vessel

$V_2 = \text{Volume of vessel} = \frac{2}{3}\pi R^3$

Clearly, $V_1 = V_2$

$\Rightarrow \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2) = \frac{2}{3}\pi R^3$ or $\frac{\overset{8}{\cancel{16}}}{\cancel{3}}\cancel{\pi}(14^2 + 7^2 + 98) = \frac{\cancel{2}}{\cancel{3}}\cancel{\pi}R^3$

$\Rightarrow R^3 = 8 + (196 + 49 + 98)$

$\Rightarrow R^3 = 8 \times 343$

$\Rightarrow R = 2 \times 7 = 14$ cm

$\therefore$ Internal diameter of the hemispherical vessel = 2R = 28 cm.

**35.** We have total number of girls 50.

So, $2 + f_1 + 12 + f_2 + 8 = 50$

$\Rightarrow f_1 + f_2 = 50 - 22 = 28$ ...(1)

For median, we make the following table :

| Height (in cm) (Class interval) | No. of girls (Frequency) | Cumulative frequency |
|---|---|---|
| 120 – 130 | 2 | 2 |
| 130 – 140 | $f_1$ | $2 + f_1$ |
| 140 – 150 | 12 | $14 + f_1$ |
| 150 – 160 | $f_2$ | $14 + f_1 + f_2$ |
| 160 – 170 | 8 | $22 + f_1 + f_2$ |
| Total | 50 | |

The median is 151.5. So, the median class is 150-160.

So, $l = 150$, N = 50, $cf = 14 + f_1$, $f = f_2$ and $h = 10$.

So, $$\text{median} = 150 + \left[\frac{\frac{50}{2} - (14 + f_1)}{f_2}\right] \times 10$$

$$\Rightarrow \quad 151.5 = 150 + \left(\frac{25 - 14 - f_1}{f_2}\right) \times 10$$

$$\Rightarrow \quad 151.5 - 150 = \frac{11 - f_1}{f_2} \times 10.$$

$$\Rightarrow \quad 1.5 \times f_2 = 110 - 10f_1.$$

$$\Rightarrow \quad 10f_1 + 1.5f_2 = 110 \qquad ...(2)$$

From (2) and (1), we get

$$10f_1 + 1.5f_2 = 110$$

$$\underset{-}{1.5f_1} + \underset{-}{1.5f_2} = \underset{-}{42} \qquad \text{[Multiplying (1) by 1.5]}$$

$$\Rightarrow \quad 8.5f_1 = 68$$

$$\Rightarrow \quad f_1 = \frac{68}{8.5} = \frac{680}{85} = \frac{40}{5} = 8.$$

Putting $f_1 = 8$ in equation (1), we get

$$8 + f_2 = 28.$$

$$\Rightarrow \quad f_2 = 28 - 8 = 20.$$

Hence, $f_1 = 8$ and $f_2 = 20$.

**36.** (*i*) We have : $a + 2d = 6000$ and $a + 6d = 7000$

So, we have $3a + 6d = 18000$ and $a + 6d = 7000$

$\Rightarrow$ $2a = 11000$ $\Rightarrow$ $a = 5500$ units.

(*ii*) Production in the 5th year $= a + 4d + 4x = 5500 + 4 \times 250 = 6500$ units.

(*iii*) Production in 6th year $= a + 5d = 5500 + 5 \times 250 = 6750$ units.

***Or***

Solving $a + 2d = 6000$ and $a + 6d = 7000$

$\Rightarrow$ $4d = 1000$ $\Rightarrow$ $d = 250$.

So, fixed increasing number = 250.

**37.** (*i*) Position of green flag is (3, 25).

(*ii*) Position of green red is (9, 20).

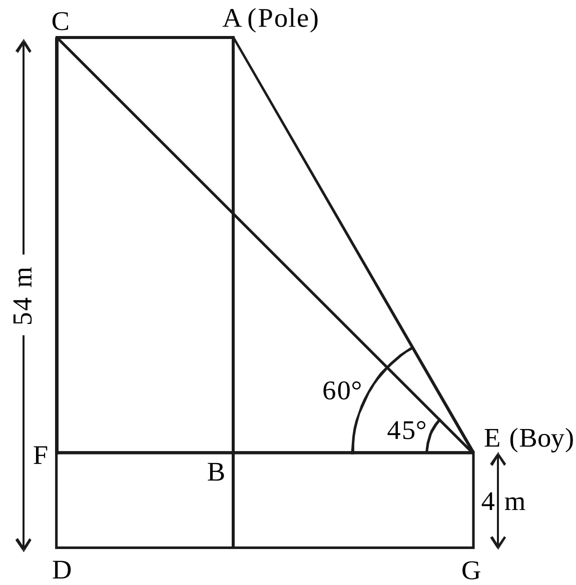
(*iii*) Distance between both the flags $= \sqrt{(9-3)^2 + (20-25)^2} = \sqrt{36+25} = \sqrt{61}$ units.

***Or***

Required point $= \left(\frac{1 \times 9 + 3 \times 3}{4}, \frac{1 \times 20 + 3 \times 25}{4}\right) = \left(\frac{18}{4}, \frac{95}{4}\right) = \left(\frac{9}{2}, \frac{95}{4}\right)$.

**38.** (*i*) From $\Delta ABE$, $\frac{BE}{AB} = \cot 60° \Rightarrow \frac{BE}{54-4} = \frac{1}{\sqrt{3}} \Rightarrow BE = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3}$ m.

So, distance of boy from the pole is $\frac{50\sqrt{3}}{3}$ m.




(*ii*) From $\Delta ABE$, $\frac{AB}{AE} = \sin 60° \Rightarrow \frac{50}{AE} = \frac{\sqrt{3}}{2} \Rightarrow AE = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$ m.

Thus, distance of first position of pigeon from the boy is $\frac{100\sqrt{3}}{3}$ m.

(*iii*) From $\Delta CFE$, $\frac{CF}{EF} = \tan 45° \Rightarrow \frac{50}{EF} = 1 \Rightarrow EF = 50$ m.

So, $AC = BF = EF - BE$

$$= 50 - \frac{50\sqrt{3}}{3} = \left(\frac{150 - 50\sqrt{3}}{3}\right) \text{ m}$$

Hence, distance travelled by the pigeon in 8 seconds is $\left(\frac{150 - 50\sqrt{3}}{3}\right) = \frac{50(3-\sqrt{3})}{3}$ m.

***Or***

From $\Delta CFE$, $\frac{CF}{CE} = \sin 45° \Rightarrow \frac{50}{CE} = \frac{1}{\sqrt{2}} \Rightarrow CE = 50\sqrt{2}$ m.

So, distance of second position of pigeon from the eye of the boy is $50\sqrt{2}$ m.

C.P. Digest

# Mathematics

## (Standard)

## Class-X

## Sample Question Paper 5

## (Answers)

**1.** (A) $2 \times \frac{1}{\sqrt{2}}$

**2.** (B) No, because if $9 + \sqrt{2} = \frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$, then $\sqrt{2} = \frac{9b + a}{b}$, but $\sqrt{2}$ is an irrational number. So, $9 + \sqrt{2} \neq \frac{a}{b}$.

**3.** (D) $x^2 + \left(\frac{3}{5}\right)x + \left(\frac{3}{10}\right)^2 - \frac{9}{100} - \frac{1}{5} = 0$

**4.** (A) $a = 2$, $b = 5$

**5.** (A) $\frac{4b}{a}$ and $-\frac{b}{a}$

**6.** (A) 120 feet

**7.** (D) 2 : 3

**8.** (D) (6, 12)

**9.** (C) 60°

**10.** (C) 15°

**11.** (B) 15°

**12.** (D) 15°

**13.** (B) 6 cm

**14.** (D) $64\pi$

**15.** (A) 952875

**16.** (C) 10

**17.** (C) 20

**18.** (A) The probability of selecting a student who prefer tennis is more than that of selecting a student who prefer cricket.

**19.** (C) (A) is true but (R) is false.

**20.** (B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

**21.** Given pair of linear equations is $y - x = 6$ or $y - x - 6 = 0$

and $3kx + 2y = 7$ or $3kx + 2y - 7 = 0$.

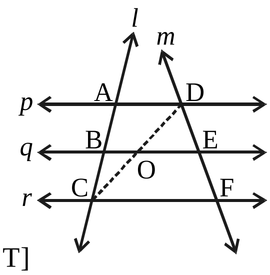
We have : $a_1 = -1$, $b_1 = 1$, $c_1 = -6$

and $a_2 = 3k$, $b_2 = 2$, $c_2 = -7$.

For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{-1}{3k} \neq \frac{1}{2} \Rightarrow 3k \neq -2$ or $k \neq \frac{-2}{3}$.

**22.** ***Hint :*** After joining CD, BPT shall be used in two triangles.

**Given :** In the given figure, $p \parallel q \parallel r$.




**To prove :** $\frac{AB}{BC} = \frac{DE}{EF}$.

**Construction :** Join CD which intersects BE at O.

**Proof :** In $\Delta$ACD, BO $\parallel$ AD.

So, $\frac{AB}{BC} = \frac{DO}{OC}$ ...(*i*) [By BPT]

Now, in $\Delta$DCF, OE $\parallel$ CF.

So, $\frac{DE}{EF} = \frac{DO}{OC}$ ...(*ii*) [By BPT]

From (*i*) and (*ii*), we get

$$\frac{AB}{BC} = \frac{DE}{EF}, \text{ Proved.}$$

***Or***

***Hint :*** BPT will be used in two triangles.

In $\Delta$ABC, we have :

DE $\parallel$ AC (Given)

So, $\frac{BE}{EC} = \frac{BD}{DA}$ (By BPT) ...(1)

Also, in $\Delta ABP$, we have :

$$DC \parallel AP \qquad \text{(Given)}$$

So, $\frac{BD}{DA} = \frac{BC}{CP}$ (By BPT) ...(2)

From (1) and (2),

$$\frac{BE}{EC} = \frac{BC}{CP} \qquad \left(\text{Each equal to } \frac{BD}{DA}\right)$$

**23.** ***Hint :*** Exterior angle property for $\Delta AXY$ and the property of isosceles triangle OAY will be used to prove the result.

From $\Delta AXY$, $\angle OAY = \angle BXY + \angle AYX$ (Exterior angle property)

$= b + a.$

But, $OA = OY$ (Radii of a circle)

So, $\angle OYA = \angle OAY$

$\Rightarrow$ $\angle OYA = b + a.$

Also, $\angle OYX = 90°$ (Angle between radius and tangent)

$\Rightarrow$ $\angle AYX + \angle OYA = 90°.$

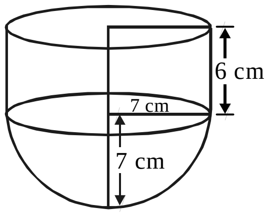
$\therefore$ $a + a + b = 90° \Rightarrow b + 2a = 90°.$

**24.** $\cos 0° = 1$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\cos 45° = \frac{1}{\sqrt{2}}$,

$\cos 60° = \frac{1}{2}$ and $\cos 90° = 0$.

Value of $\cos\theta$ decreases from 1 to 0 as $\theta$ increases from 0° to 90°.

**25.** Let $h$ be the height and $r$ be the base radius of the cylinder.




So, $r = 7$ cm and $h = 13 - 7 = 6$ cm.

Therefore, inner surface area of the vessel

= Curved surface area of hemisphere + Curved surface area of cylinder

$= 2\pi r^2 + 2\pi rh = 2\pi r[r + h].$

$= 2 \times \frac{22}{7} \times 7[7 + 6]\text{ cm}^2.$

$= 44 \times 13 \text{ cm}^2 = 572 \text{ cm}^2.$

***Or***

Length of the cuboid $5 + 5 = 10$ cm,

breadth of the cuboid = 5 cm

and height of the cuboid = 5 cm.

So, surface area of the cuboid $= 2(lb + bh + hl).$

$= 2(10 \times 5 + 5 \times 5 + 5 \times 10)\text{ cm}^2.$

$= 2(125) = 250 \text{ cm}^2.$

**26.** ***Hint :*** By contradiction

Let us suppose that $5 + 3\sqrt{2}$ is a rational number in the form of $\frac{p}{q}$, where $p$ and $q$ are integers such that $q \neq 0$.

Then, $5 + 3\sqrt{2} = \frac{p}{q}.$

$\Rightarrow$ $3\sqrt{2} = \frac{p}{q} - 5$

$\Rightarrow$ $\sqrt{2} = \frac{p - 5q}{3q}.$

Here, on the right hand side, $p - 5q$ and $3q$ are integers and, therefore, $\frac{p-5q}{3q}$ is a rational number.

But, it is given that $\sqrt{2}$ is an irrational number.

So, the two sides cannot be equal. This contradiction is because of our assumption that $5+3\sqrt{2}$ is a rational number.

Hence, $5+3\sqrt{2}$ is an irrational number.

**27.** ***Hint :*** Sum of the zeroes $\frac{-b}{a}$ and product of zero $=\frac{c}{a}$ will be used.

Let one zero of $2x^2 - 5x - (2k + 1)$ be $\alpha$.

Then, the other zero is $2\alpha$.

Now, we have : $\alpha + 2\alpha = -\frac{(-5)}{2} = \frac{5}{2}$

$\Rightarrow \quad 3\alpha = \frac{5}{2} \quad \Rightarrow \quad \alpha = \frac{5}{6}.$

So, zeroes are $\frac{5}{6}$ and $2 \times \frac{5}{6}$, *i.e.*, $\frac{5}{6}$ and $\frac{5}{3}$.

Again, product of zeroes $= \frac{5}{6} \times \frac{5}{3} = \frac{c}{a}$.

$\Rightarrow \quad \frac{25}{18} = \frac{-(2k+1)}{2}$

$\Rightarrow \quad \frac{25}{9} = -(2k+1)$

$\Rightarrow \quad 25 = -18k - 9$

$\Rightarrow \quad 18k = -34$

$\Rightarrow \quad k = \frac{-34}{18} = -\frac{17}{9}.$

Hence, the zeroes are $\frac{5}{6}$ and $\frac{5}{3}$ and the value of $k$ is $-\frac{17}{9}$.

**28.** Let the unit digit be $x$ and the tens digit be $y$.

As per given conditions, we have :

$$x + y = 9 \quad \text{...(1)}$$

The number is $10y + x$. After reversing the order of digits number became $10x + y$.

As per condition,

$$9(10y + x) = 2(10x + y)$$

$\Rightarrow \quad 90y + 9x = 20x + 2y$

$\Rightarrow \quad 11x - 88y = 0$

$\Rightarrow \quad x - 8y = 0 \quad \text{...(2)}$

Subtracting (2) from (1), we get

$$y + 8y = 9 - 0$$

$\Rightarrow \quad 9y = 9 \quad \Rightarrow \quad y = \frac{9}{9} = 1.$

So, from (1), we get $x = 8$.

Thus, the number is 18.

***Or***

***Hint :*** The two equations will be added and subtracted to obtain two new equations. Then, these two new equations will be solved in usual manner.

We have :

$$49x + 51y = 499 \quad ...(1)$$
$$51x + 49y = 501 \quad ...(2)$$

Adding (1) and (2), we get

$$(49 + 51)x + (51 + 49)y = 499 + 501$$
$$\Rightarrow \quad 100x + 100y = 1000$$
$$\Rightarrow \quad x + y = 10 \quad ...(3)$$

Subtracting (1) from (2), we get

$$51x - 49x + 49y - 51y = 501 - 499$$
$$\Rightarrow \quad 2x - 2y = 2$$
$$\Rightarrow \quad x - y = 1 \quad ...(4)$$

Adding (3) and (4), we get

$$2x = 11 \quad \Rightarrow \quad x = \frac{11}{2}.$$

Putting $x = \frac{11}{2}$ in (3), we get

$$\frac{11}{2} + y = 10$$
$$\Rightarrow \quad y = 10 - \frac{11}{2} = \frac{9}{2}.$$

Thus, $x = \frac{11}{2}$ and $y = \frac{9}{2}$.

**29.** ***Hint :*** Property of isosceles triangle will be used and then required angles shall be determined using angle sum property of a triangle.

From the figure, $PR = PQ$

So, $\angle PQR = \angle PRQ = 70°$. (Angles opposite the equal sides of $\Delta PQR$)

Now, $\angle QPR + \angle PQR + \angle PRQ = 180°$.

So, $\angle QPR + 70° + 70° = 180°$

$\Rightarrow \quad \angle QPR = 180° - 140° = 40°$.

Now, $\angle OQP = 90°$. (Angle between tangent and radius)

So, $\angle OQR = \angle OQP - \angle PQR$

$= 90° - 70° = 20°$.

**30.** ***Hint :*** $\tan A = \frac{1}{\sqrt{3}}$ gives $A = 30°$. Then using sum of angles property, $\angle B$ will be found.

We have :

$$\tan A = \frac{1}{\sqrt{3}}.$$
$$\Rightarrow \quad \tan A = \tan 30°$$
$$\Rightarrow \quad A = 30°.$$

But in $\Delta ABC$,

$$\angle A + \angle B + \angle C = 180°.$$

So, $30° + \angle B + 90° = 180°$ [$\because$ $\angle C = 90°$, given]

$\Rightarrow \quad \angle B = 180° - 120° = 60°$.

So, $\sin A \cos B + \cos A \sin B = \sin 30° . \cos 60° + \cos 30° . \sin 60°$

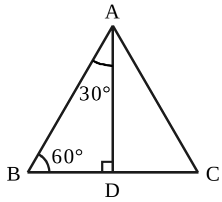
$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$
$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1.$$

Hence, proved.

***Or***

Let ABC be an equilateral triangle of side $2a$.

Draw AD $\perp$ BC (see figure).




Now, $\Delta ABD \cong \Delta ACD$ (By RHS)

So, $BD = DC$. (CPCT)

$\therefore$ $\angle BAD = \angle CAD$

$\Rightarrow$ $\angle BAD = \frac{1}{2} \times 60° = 30°$.

Now, $BD = DC$.

$\Rightarrow$ $BD = \frac{1}{2}BC = \frac{1}{2} \times 2a = a$.

Also, $AD^2 = AB^2 - BD^2$.

$\Rightarrow$ $AD^2 = (2a)^2 - a^2 = 3a^2$.

$\Rightarrow$ $AD = a\sqrt{3}$.

Now, from rt. triangle ABD, we get

$$\tan 30° = \frac{BD}{AD} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

**31.** ***Hint :*** First, all the possible outcomes shall be listed and then the probabilities shall be found for each part (*a*), (*b*) and (*c*).

The total outcomes on tossing three unbiased coins together are

HHH, HHT, HTH, THH, TTH, THT, HTT, TTT.

So, their number = 8.

(*a*) Let the event of getting two heads be E.

Then, outcomes favourable to E are : HHT, HTH, THH.

$\Rightarrow$ Number of outcomes favourable to E = 3.

So, $P(E) = \frac{3}{8}$.

(*b*) Let the event of getting at least two heads be F.

Then, outcomes favourable to F are : HHT, HTH, THH, HHH.

$\Rightarrow$ Number of outcomes favourable to F = 4.

So, $P(F) = \frac{4}{8} = \frac{1}{2}$.

[***Check :*** May write $\frac{7}{8}$]

(*c*) Let G be the event of getting at most two heads.

Then, outcomes favourable to G are : TTT, HTT, THT, HTT, HHT, HTH, THH.

$\Rightarrow$ Number of outcomes favourable to G = 7.

So, $P(G) = \frac{7}{8}$.

[***Check :*** May write $\frac{4}{8}$]

**32.** Tables for the given equations are :

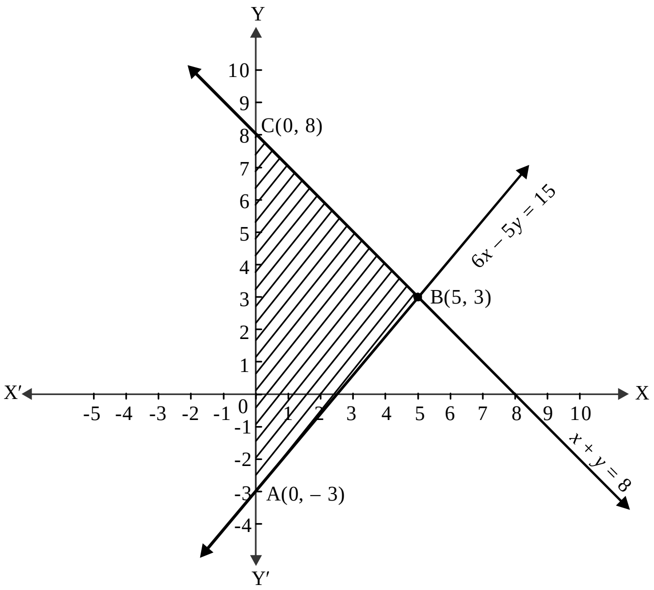
$x + y = 8$ :

| | | | |
|---|---|---|---|
| $x$ | 0 | 8 | 2 |
| $y$ | 8 | 0 | 6 |

and $6x - 5y = 15$ :

| | | | |
|---|---|---|---|
| $x$ | 0 | 2.5 | 1 |
| $y$ | – 3 | 0 | – 1.8 |

Graph is as shown below :




$\Delta$ABC is the required shaded region.

Vertices of $\Delta$ABC are A(0, – 3), B(5, 3) and C(0, 8).

***Or***

***Hint :*** Both the polynomial will be factorised and common factor will give the answer.

$$x^2 - 3x + 2 = 0$$
$$\Rightarrow \quad x^2 - 2x - x + 2 = 0$$
$$\Rightarrow \quad x(x-2) - 1(x-2) = 0$$
$$\Rightarrow \quad (x-2)(x-1) = 0$$
$$\Rightarrow \quad (x-2) = 0 \text{ or } (x-1) = 0$$
$$\Rightarrow \quad x = 2 \text{ or } x = 1 \quad ...(1)$$

Also,
$$x^2 - 6x + 5 = 0$$
$$\Rightarrow \quad x^2 - 5x - x + 5 = 0$$
$$\Rightarrow \quad x(x-5) - 1(x-5) = 0$$
$$\Rightarrow \quad (x-5)(x-1) = 0$$
$$\Rightarrow \quad (x-5) = 0 \text{ or } (x-1) = 0$$
$$\Rightarrow \quad x = 5 \text{ or } x = 1 \quad ...(2)$$

From (1) and (2) above, it can be said that $x^2 - 3x + 2$ and $x^2 - 6x + 5$ will both become zero at $x = 1$.

**33.** AB $\perp$ BD and XY $\perp$ BD (∠ABD = 90°, ∠XYD = 90°)

$\Rightarrow$ AB || XY

So, ∠BAX = ∠YXD (Corresponding angles)

Hence, $\Delta$DXY ~ $\Delta$DAB (By AA similarity criterion)

So, $\dfrac{DY}{DB} = \dfrac{c}{a} = \dfrac{DX}{DA}$ ...(1) (Corollary to BPT)

Also, by AA similarity criterion,

$\Delta$BXY ~ $\Delta$BCD

So, $\dfrac{BY}{DB} = \dfrac{c}{b} = \dfrac{BX}{BC}$ ...(2)

From (1), $\dfrac{DY}{DB} = \dfrac{c}{a} \Rightarrow 1 - \dfrac{DY}{DB} = 1 - \dfrac{c}{a}$.

$\Rightarrow \quad \dfrac{DB - DY}{DB} = \dfrac{a - c}{a}$

$\Rightarrow \quad \dfrac{BY}{DB} = \dfrac{a - c}{a}.$

So, from (2), we have :

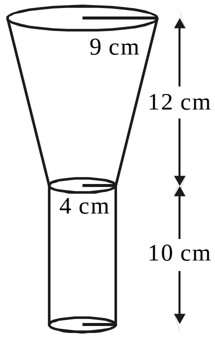
$\dfrac{a - c}{a} = \dfrac{c}{b}.$

$\Rightarrow \quad ab - bc = ac$

$\Rightarrow \quad ab = ac + bc$

$\Rightarrow \quad ab = c\,(a + b)$, **proved.**

**34.** Let the internal radius of the sphere be $r$ cm.




So, volume of hollow sphere = Volume of the cone

$\Rightarrow \quad \dfrac{4}{3}\pi\,(r_1^3 - r_2^3) = \dfrac{1}{3}\pi R^2 h$

$\Rightarrow \quad \dfrac{4}{3}\pi\,(5^3 - r^3) = \dfrac{1}{3}\pi \times 7^2 \times 8 \quad \Rightarrow \quad 4\,(125 - r^3) = 392$

$\Rightarrow \quad 500 - 4r^3 = 392$

$\Rightarrow \quad 4r^3 = 500 - 392 \quad \Rightarrow \quad 4r^3 = 108$

$r^3 = \dfrac{108}{4} = 27 \quad \Rightarrow \quad r = 3$ cm

So, diameter = 2 × 3 cm = 6 cm

***Or***

For frustum, $r_1 = 4$ cm, $r_2 = 9$ cm and $h = 12$ cm

So, $l = \sqrt{(r_2 - r_1)^2 + h^2} = \sqrt{(9-4)^2 + 12^2} = \sqrt{25 + 144} = 13$ cm

Tin required for funnel = Curved surface area of cylinder + Curved surface area of frustum

$= 2\pi r_1 h + \pi\,(r_1 + r_2)\,l$

$= \pi\,[(2 \times 4 \times 10 + (4 + 9) \times 13)]$

$= \pi\,(80 + 169)$

$= \pi \times 249 = 249\pi$ cm$^2$

**35.** The total number of females is 50.

So, $4 + 5 + f_1 + f_2 + 9 + 7 + 1 = 50.$

$\Rightarrow \quad f_1 + f_2 = 50 - 26 = 24 \qquad ...(1)$

For median of the given data, we make the following table :

| Number of heart beats per minute (Class interval) | No. of females (Frequency) | Cumulative frequency |
|---|---|---|
| 64-68 | 4 | 4 |
| 68-72 | 5 | 9 |
| 72-76 | $f_1$ | $9 + f_1$ |
| 76-80 | $f_2$ | $9 + f_1 + f_2$ |
| 80-84 | 9 | $18 + f_1 + f_2$ |
| 84-88 | 7 | $25 + f_1 + f_2$ |
| 88-92 | 1 | $26 + f_1 + f_2$ |
| Total | 50 | |

The median is 78.

So, the median class is 76 – 80.

Here, $l = 76$, N = 50, $f = f_2$, $cf = 9 + f_1$ and $h = 4$.

So, $$\text{median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h.$$

$$= 76 + \left[\frac{\frac{50}{2} - (9 + f_1)}{f_2}\right] \times 4.$$

$$\Rightarrow \quad 78 = 76 + \left[\frac{25 - 9 - f_1}{f_2}\right] \times 4.$$

$$\Rightarrow \quad 78 - 76 = \frac{16 - f_1}{f_2} \times 4.$$

$$\Rightarrow \quad \frac{2}{4} = \frac{16 - f_1}{f_2}.$$

$$\Rightarrow \quad f_2 = 32 - 2f_1$$

$$\Rightarrow \quad 2f_1 + f_2 = 32 \qquad \text{...(2)}$$

From (2) and (1), we get

$$2f_1 + f_2 = 32$$
$$f_1 + f_2 = 24$$
$$- \quad - \quad -$$

$$\Rightarrow \quad f_1 = 8$$

Putting $f_1 = 8$ in equation (1), we get

$$8 + f_2 = 24.$$

$$\Rightarrow \quad f_2 = 24 - 8 = 16.$$

Hence, $f_1 = 8$ and $f_2 = 16$.

**36.** (*i*) AP is 4, 6, 8, ...; $a = 4$, $d = 2$.

(*ii*) AP is 12, 19, 26, ...; $a = 12$, $d = 7$.

(*iii*) $n$th term of AP 4, 6, 8, ... is $4 + (n - 1) \times 2 = 2n + 2$.

***Or***

Let $n$th term of AP 12, 19, 26, ...; 6, 61.

So, $12 + (n - 1) \times 7 = 61 \Rightarrow (n - 1) = \frac{49}{2} \Rightarrow 7 \Rightarrow n = 8$.

So, 8th figure will have 61 sticks.

**37.** (*i*) Coordinates of H1 are (3, 3).

(*ii*) Coordinates of Police Station are (5, 5).

(*iii*) Coordinates of H3 are (2, 2).

Let the ratio be $k : 1$.

So, $3 = \frac{k \times 5 + 1 \times 2}{(k + 1)} \Rightarrow 3k + 3 = 5k + 2 \Rightarrow 2k = 1 \Rightarrow k = \frac{1}{2}$.

***Or***

Coordinates of pond are (3, 5).

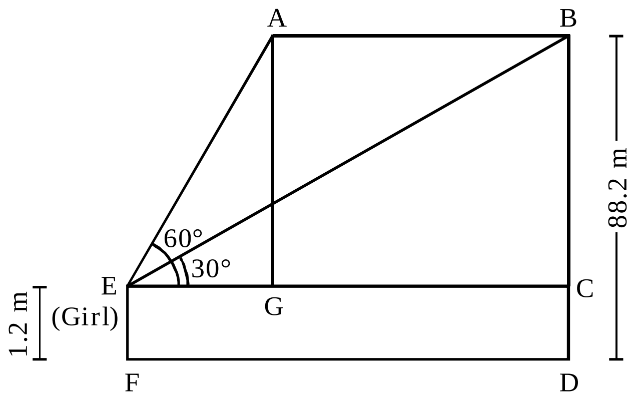
Coordinates of hospital are (5, 3).

So, $x$-coordinates of well = $\dfrac{4 \times 5 + 1 \times 3}{4 + 1} = \dfrac{23}{5}$.

**38.** ($i$) From ΔAEG, $\dfrac{AG}{AE} = \sin 60° \Rightarrow \left(\dfrac{88.2 - 1.2}{AE}\right) = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \dfrac{87}{AE} = \dfrac{\sqrt{3}}{2} \Rightarrow AE = \dfrac{87 \times 2}{\sqrt{3}} = 58\sqrt{3}$ m.

Thus, distance of first position of balloon from the eye of the girl is $58\sqrt{3}$ m.




($ii$) From ΔBEC, $\dfrac{BC}{BE} = \sin 30° \Rightarrow \dfrac{87}{BE} = \dfrac{1}{2} \Rightarrow BE = 87 \times 2 \text{ m} = 174$ m .

($iii$) From ΔAEG, $\dfrac{AG}{EG} = \tan 60° \Rightarrow \dfrac{87}{EG} = \sqrt{3} \Rightarrow EG = 29\sqrt{3}$ m ...(1)

From ΔBEC, $\dfrac{BC}{EC} = \tan 30° \Rightarrow \dfrac{87}{EC} = \dfrac{1}{\sqrt{3}} \Rightarrow EC = 87\sqrt{3}$ m ...(2)

Distance travelled by the balloon during the interval

$= AB = CG = 87\sqrt{3} \text{ m} - 29\sqrt{3} = 58\sqrt{3}$ m.

***Or***

Speed of the balloon $= \dfrac{AB}{30}$ metres per second.

$= \dfrac{58\sqrt{3}}{30}$ metres per second.

$= \dfrac{29\sqrt{3}}{15}$ metres per second.

C.P. Digest

# Mathematics

*(Standard)*

## Class-X

## Sample Question Paper 6

## (Answers)

**1.** (C) An irrational number

**2.** (A) A rational number

**3.** (A) On substituting $x = 2$ and $x = 3$ on the left-hand side of the equation, the result should be 0

**4.** (D) $y = 3 - 2a$

**5.** (C) The student calculated the roots of the equation that can be obtained by adding 1 to the equation that the student solved.

**6.** (B) 79° **7.** (C) $\Delta BAC \sim \Delta DEF$ **8.** (B) 500 m **9.** (B) 40°

**10.** (B) JL **11.** (A) $\angle ARC$ **12.** (B) $25(\sqrt{3} + 1)$ m

**13.** (C) $25/\pi$ cm **14.** (C) $\frac{200}{\pi}$ cm **15.** (D) 2.1 cm

**16.** (D) Mean = Median = Mode **17.** (D) 11.2 **18.** (C) $\frac{3}{26}$

**19.** (A) Both (A) and (R) are true and (R) is the correct explanation of (A).

**20.** (D) (R) is true but (A) is false.

**21.** Given pair of linear equations is

$$y - x = 1$$

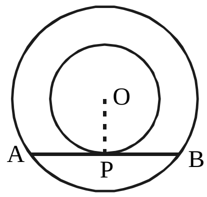
and $$x - 3y = 4.$$

Now, $a_1 = -1$, $b_1 = 1$, $c_1 = -1$ and $a_2 = 1$, $b_2 = -3$, $c_2 = -4$.

Then, $\frac{a_1}{a_2} = \frac{-1}{1} = -1$, $\frac{b_1}{b_2} = \frac{1}{-3}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so system is consistent and has a unique solution.

**22.** In the figure, chord AB of the larger concentric circle with centre O touches smaller circle at P. Join OP.

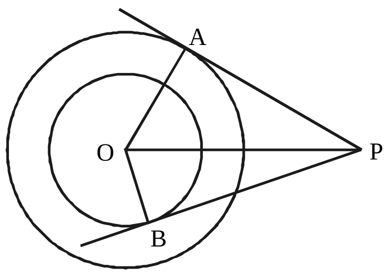



Now, OP $\perp$ AB (Tangent and radius are perpendicular)

So, AB is bisected at P, because perpendicular from the centre of a circle to a chord, bisects the chord, proved.

***Or***

***Hint :*** Pythagoras Theorem will be used for two right triangles.



From rt. $\Delta$OAP,

$$OP^2 = OA^2 + PA^2$$
$$= 5^2 + 12^2 = 169.$$

$\Rightarrow$ $OP = \sqrt{169} = 13$ cm.

Now, from rt. $\Delta$OBP,

$$PB^2 = OP^2 - OB^2 = 13^2 - 3^2.$$

$\Rightarrow$ $PB^2 = 169 - 9 = 160.$

$\Rightarrow$ $PB = \sqrt{160} = \sqrt{4 \times 4 \times 10}$

$$= 4\sqrt{10} \text{ cm}.$$

**23.** ***Hint :*** In case of tangents from an external point, the line joining centre to the external point is equally inclined to the corresponding radii. This fact will be utilised in four pairs of such tangents.

AP and AS are tangents from external point A.

So, $\angle AOP = \angle AOS$ ...(1)

Similarly, $\angle BOP = \angle BOQ$, ...(2)

$\angle COR = \angle COQ$ ...(3)

and $\angle DOR = \angle DOS$ ...(4)

Adding, we get

$$\angle AOP + \angle BOP + \angle COR + \angle DOR = \angle AOS + \angle BOQ + \angle COQ + \angle DOS$$

$\Rightarrow$ $\angle AOB + \angle COD = \angle AOD + \angle BOC$ ...(5)

But, $\angle AOB + \angle COD + \angle AOD + \angle BOC = 360°$.

So, from eqn. (5),

$$\angle AOB + \angle COD + \angle AOD + \angle BOC = \angle AOB + \angle COD + \angle AOB + \angle COD$$
$$= 360°.$$

$\Rightarrow$ $2(\angle AOB + \angle COD) = 360°.$

$\Rightarrow$ $\angle AOB + \angle COD = \frac{1}{2} \times 360° = 180°.$

**24.** $\sin(A + B) = 1 = \sin 90°.$

$\Rightarrow$ $A + B = 90°$ ...(1)

Also, $\sin(A - B) = \frac{1}{2} = \sin 30°.$

$\Rightarrow$ $A - B = 30°$ ...(2)

Adding (1) and (2),

$$2A = 120°$$

$\Rightarrow$ $A = \frac{120°}{2} = 60°.$

Putting A = 60° in (1), we get

$$60° + B = 90°$$

$\Rightarrow$ $B = 90° - 60° = 30°.$

**25.** Let the side of the new cube be '$a$'.

So, $\sqrt{a^2 + a^2 + a^2} = 12\sqrt{3}.$

$\Rightarrow$ $a = 12.$

Let the edges of three cubes be $3x$, $4x$ and $5x$.

Side of new cube = 12 cm.

$\therefore$ Volume of three cubes = Volume of single cube

$\Rightarrow$ $(3x)^3 + (4x)^3 + (5x)^3 = 12 \times 12 \times 12.$

$\Rightarrow$ $27x^3 + 64x^3 + 125x^3 = 12 \times 12 \times 12$

$\Rightarrow \quad 216x^3 = 12 \times 12 \times 12.$

$\Rightarrow \quad x^3 = \dfrac{12 \times 12 \times 12}{216}.$

$= \dfrac{12 \times 12 \times 12}{6 \times 6 \times 6} = \left(\dfrac{12}{6}\right)^3.$

$\Rightarrow \quad x = \dfrac{12}{6} = 2.$

$\therefore$ Edges of three cubes are 6 cm, 8 cm and 10 cm.

***Or***

Let the height of the cone be $h$ cm.

Now, volume of the cone = Volume of the cylinder.

$\Rightarrow \quad \dfrac{1}{3}\pi \times 14 \times 14 \times h = \pi \times 14 \times 14 \times 20.$

$\Rightarrow \quad h = \dfrac{\pi \times 14 \times 14 \times 20 \times 3}{\pi \times 14 \times 14}$

$= 60$ cm

**26.** ***Hint :*** By contradiction

Let us suppose that $\sqrt{3} + \sqrt{5}$ is rational.

Then, we can find some integers $p$ and $q$ such that $\sqrt{3} + \sqrt{5} = \dfrac{p}{q}$, where $q \neq 0$.

Squaring both the sides, we get

$$\left(\sqrt{3} + \sqrt{5}\right)^2 = \left(\frac{p}{q}\right)^2$$

or $\quad \left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2 + 2\sqrt{3} \times \sqrt{5} = \dfrac{p^2}{q^2}$

or $\quad 3 + 5 + 2\sqrt{15} = \dfrac{p^2}{q^2}$

or $\quad \sqrt{15} = \dfrac{1}{2}\left(\dfrac{p^2}{q^2} - 8\right)$

On the right hand side, $p$ and $q$ are integers. So, $\dfrac{1}{2}\left(\dfrac{p^2}{q^2} - 8\right)$ is a rational number.

But, we know that $\sqrt{15}$ is an irrational number. This contradicts the above equality.

This contradiction is because of our incorrect assumption that $\sqrt{3} + \sqrt{5}$ is rational.

Hence, $\sqrt{3} + \sqrt{5}$ is irrational.

**27.** We have :

$$t^2 + 8t + 16 = t^2 + 4t + 4t + 16$$
$$= t(t + 4) + 4(t + 4)$$
$$= (t + 4)(t + 4).$$

So, the zeroes are given by

$$t + 4 = 0 \text{ and } t + 4 = 0.$$

$\Rightarrow \quad t = -4$ and $t = -4$.

Thus, zeroes are $-4$ and $-4$.

**Verification :**

Sum of the zeroes $= -4 + (-4) = -8$

$= \dfrac{-(8)}{1} = \dfrac{-b}{a}.$

$$\text{Product of zeroes} = -4 \times (-4) = 16$$

$$= \frac{16}{1} = \frac{c}{a}.$$

**28.** Let the monthly income of the man be ₹ $x$ and that of his wife be ₹ $y$.

As per given conditions, we have :

$$x - y = 600 \quad ...(1)$$

$$\frac{x}{10} + \frac{y}{6} = 1500$$

or $$\frac{3x + 5y}{30} = 1500$$

$\Rightarrow$ $$3x + 5y = 45000 \quad ...(2)$$

Multiplying (1) by 5, we get

$$5x - 5y = 3000 \quad ...(3)$$

Adding (2) and (3), we get

$$8x = 48000$$

$\Rightarrow$ $$x = \frac{48000}{8} = 6000.$$

Putting $x = 6000$ in (1), we get

$$6000 - y = 600$$

$\Rightarrow$ $$y = 6000 - 600 = 5400.$$

Thus, man's monthly income is ₹ 6000 and wife's income = ₹ 5400.

***Or***

Let the present ages (in years) of father and son be $x$ and $y$ respectively.

As per given conditions, we have :

$$x + y = 40 \quad ...(1)$$

and $$x = 3y \quad ...(2)$$

From eqns. (1) and (2), we get

$$3y + y = 40$$

$\Rightarrow$ $$4y = 40$$

$\Rightarrow$ $$y = \frac{40}{4} = 10.$$

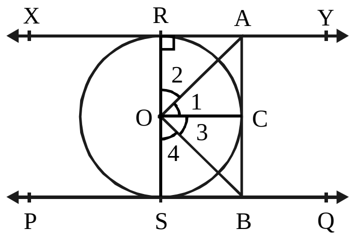
Putting $y = 10$ in eqn. (1), we get

$$x + 10 = 40$$

$\Rightarrow$ $$x = 40 - 10 = 30.$$

Thus, ages of father and son are respectively 30 years and 10 years.

**29.** XY || PQ (Given)




The line segment RS joining the points of contact of two parallel tangents passes through the centre O is the diameter.

In $\Delta$ROA and $\Delta$COA,

| | | |
|---|---|---|
| | OR = OC | (Radii of the same circle) |
| | OA = OA | (Common) |
| | $\angle$ORA = $\angle$OCA | (Each 90°) |
| $\therefore$ | $\Delta$ROA $\cong$ $\Delta$COA | (RHS) |
| $\therefore$ | $\angle 1 = \angle 2$ | (By CPCT) |

Similarly, $\Delta$SOB $\cong$ $\Delta$COB and so $\angle 3 = \angle 4$.

Since ROS is diameter, therefore

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°.$$

$\Rightarrow$ $2\angle 1 + 2\angle 3 = 180°.$ $(\because \angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4)$

$\Rightarrow$ $\angle 1 + \angle 3 = \angle 1 + \angle 3 = \frac{180}{2} = 90°.$

$$2(\angle 1 + \angle 3) = 180°$$

$\Rightarrow$ $\angle AOB = 90°.$

**30.** $$\text{LHS} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} - \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2 - (\cos\theta - \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$= \frac{(\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta) - (\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta)}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{(1 + 2\cos\theta\sin\theta) - (1 - 2\cos\theta\sin\theta)}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{1 + 2\cos\theta\sin\theta - 1 + 2\cos\theta\sin\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{4\cos\theta\sin\theta}{\cos^2\theta - \sin^2\theta} = \frac{\frac{4\cos\theta\sin\theta}{\cos\theta\sin\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta}}$$

[Dividing the numerator and denominator by $\cos\theta \sin\theta$]

$$= \frac{4}{\cot\theta - \tan\theta}, \text{ proved.}$$

$$= \frac{4}{\frac{1}{\tan\theta} - \tan\theta} = \frac{4}{\frac{1 - \tan^2\theta}{\tan\theta}} = 4 \times \frac{\tan\theta}{1 - \tan^2\theta}$$

$$= \frac{4\tan\theta}{1 - \tan^2\theta} = \text{RHS.}$$

Hence, proved.

***Or***

$$\text{LHS} = \tan\theta - \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta - (1 - \sin^2\theta)}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta - 1 + \sin^2\theta}{\cos\theta\sin\theta} = \frac{2\sin^2\theta - 1}{\cos\theta\sin\theta}, \text{ proved.}$$

$$= \frac{2[1 - \cos^2\theta] - 1}{\cos\theta\sin\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2 - 2\cos^2\theta - 1}{\cos\theta\sin\theta} = \frac{1 - 2\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \text{RHS.}$$

Hence, proved.

**31.** ***Hint :*** For (*ii*) outcomes divisible by 5 = B5, B10, P5, P10, N5, N10, O5, O10, G5, G10.
For (*iii*) variable outcomes B11, P11, N11, G11, O11.

(*i*) Total outcomes = 65.

Favourable outcomes = 13.

So, required probabilty $= \frac{13}{65} = \frac{1}{5}$.

(*ii*) Total outcomes = 65.

Favourable outcomes = 65 – 5 × 2

= 55.

So, required probability $= \frac{55}{65} = \frac{11}{13}$.

(*iii*) Favourable outcomes 5 × 1 = 5.

So, required probability $= \frac{5}{65} = \frac{1}{13}$.

**32.** We have :

$$3x + 4y = 12$$
$$(m + n)x + 2(m - n)y = 5m - 1$$

For infinitely many solutions,

$$\frac{3}{m+n} = \frac{4}{2(m-n)} = \frac{12}{5m-1}$$

$\Rightarrow$ $$\frac{3}{m+n} = \frac{4}{2(m-n)}$$

or $6m - 6n = 4m + 4n$

or $2m = 10n$

or $m = 5n$ ...(1)

Also, $$\frac{4}{2(m-n)} = \frac{12}{5m-1}$$

or $20m - 4 = 24m - 24n$

or $24n - 4 = 24m - 20m$

or $24n - 4 = 4m$

or $6n - 1 = m$ ...(2)

From (1) and (2),

$$6n - 1 = 5n$$

or $n = 1$.

Putting $n = 1$ in equation (1), we get

$$m = 5 \times 1 = 5.$$

Hence, for infinitely many solutions of the given pair of linear equations $m = 5$ and $n = 1$.

***Or***

Let the first three terms of the AP be $a - d$, $a$ and $a + d$.

So, $(a - d) + a + (a + d) = 21$.

$\Rightarrow$ $3a = 21$

$\Rightarrow$ $a = 7$ ...(1)

Also, $(a - d)^2 + a^2 + (a + d)^2 = 155$.

$\Rightarrow$ $a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 155$

$\Rightarrow$ $3a^2 + 2d^2 = 155$.

$\Rightarrow$ $3 \times 49 + 2d^2 = 155$ [From (1)]

$\Rightarrow$ $2d^2 = 155 - 147$

$\Rightarrow \quad 2d^2 = 8$

$\Rightarrow \quad d^2 = \frac{8}{2} = 4.$

$\Rightarrow \quad d = \pm 2.$

So, the AP is 7 – 2, 7, 7 + 2, ...

or 7 + 2, 7, 7 – 2, ...

*i.e.*, 5, 7, 9, ... or 9, 7, 5, ... .

**33.** In $\Delta$EDM and $\Delta$BCM, we have :

$DM = CM$ (Given)

$\angle DME = \angle BMC$ (Vertically opposite angles)

$\angle DEM = \angle CBM$ (Alternate interior angles, DE || BC)

So, $\Delta EDM \cong \Delta BCM$ (By AAS congruence criterion)

$\Rightarrow \quad DE = BC$ (CPCT)

So, $DE = AD$ (Because BC = AD)

Now, in $\Delta$AEL and $\Delta$CBL, we have :

$\angle ELA = \angle BLC$ (Vertically opposite angles)

$\angle DEL = \angle CBL$ (Alternate interior angles)

So, $\Delta AEL \sim \Delta CBL$ (By AA similarity criterion)

$\Rightarrow \quad \frac{AE}{EL} = \frac{CB}{BL}$ (Corresponding sides are proportional)

$\Rightarrow \quad \frac{2AD}{EL} = \frac{BC}{BL}$ (Since AD = DE)

$\Rightarrow \quad \frac{2AD}{EL} = \frac{AD}{BL}$ (BC = AD)

$\Rightarrow \quad 2BL = EL \quad \Rightarrow \quad EL = 2BL$, **proved.**

**34.** $r_1$ = Radius of lower end = 1 cm

$r_2$ = Radius of upper end = 2.5 cm

and $h$ = Height of frustum of cone

So, $l = \sqrt{h^2 + (r_2 - r_1)^2}$

$= \sqrt{36 + (2.5 - 1)^2}$

$= \sqrt{38.25}$ cm = 6.18 cm

External surface area = C.S.A. of frustum + Surface area of hemisphere

$= \pi(r_1 + r_2)\, l + 2\pi r_1^2$

$= \pi(1 + 2.5) \times 6.18 + 2\pi(1)^2$

$= \frac{22}{7}[21.63 + 2]\ \text{cm}^2$

$= \left(\frac{22}{7} \times 23.63\right) \text{cm}^2$

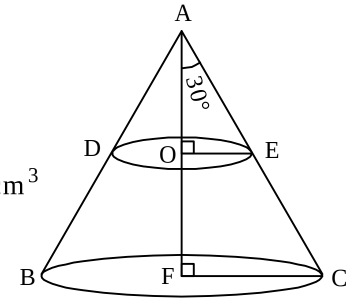
$= 74.26\ \text{cm}^2$

***Or***

In right $\Delta$AFC, $\tan 30° = \frac{FC}{AF}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{FC}{20} \quad \Rightarrow \quad FC = \frac{20}{\sqrt{3}}\ \text{cm} = r_1$ (Given : Height, AF = 20 cm)

In right triangle AOE, $\tan 30° = \frac{OE}{AO}$ (∵ The plane DOE is in the middle *i.e.*, $AO = OF = \frac{20}{2} = 10$ cm)




$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{OE}{10} \Rightarrow OE = \frac{10}{\sqrt{3}} \text{ cm} = r_2$$

$$\text{Volume of frustum} = \frac{1}{3}\pi h (r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3}\pi \times 10 \left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right) \text{cm}^3$$

(Here, $h$ = AF – AO = 20 cm – 10 cm = 10 cm)

$$= \frac{1}{3}\pi \times \frac{7000}{3} \text{ cm}^3$$

Now, let the length of the wire be $h_1$

So, $$\frac{1}{3}\pi \times \frac{7000}{3} = \pi h_1 \times \frac{1}{32} \times \frac{1}{32} \Rightarrow h_1 = \frac{7000 \times 32 \times 32}{3 \times 3}$$

Length of wire = 796444.44 cm = 7964.44 m

**35.** Let us prepare the following table :

| Class interval | Frequency | Cumulative frequency |
|---|---|---|
| 0 – 10 | 5 | 5 |
| 10 – 20 | $x$ | $5 + x$ |
| 20 – 30 | 20 | $25 + x$ |
| 30 – 40 | 15 | $40 + x$ |
| 40 – 50 | $y$ | $40 + x + y$ |
| 50 – 60 | 5 | $45 + x + y$ |
| Total | $N = 45 + x + y = 60$ | |

Here, N = 60 (Given). So, $\frac{N}{2} = 30$.

Now, the median 28.5 belongs to the class 20 – 30.

∴ Median class is 20 – 30.

Here, $l = 20$, $f = 20$, $c.f. = 5 + x$ and $h = 10$.

Now, $$\text{median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h.$$

$$\Rightarrow \quad 28.5 = 20 + \left(\frac{30 - 5 - x}{20}\right) \times 10$$

$$\Rightarrow \quad 28.5 = 20 + \frac{25 - x}{2}.$$

$$\Rightarrow \quad 57 = 40 + 25 - x.$$

$$\Rightarrow \quad 57 = 65 - x$$

$$\Rightarrow \quad x = 8.$$

Again, N = 60.

So, $45 + x + y = 60$

$\Rightarrow \quad 45 + 8 + y = 60 \quad [\because x = 8]$

$\Rightarrow \quad y = 60 - 53 = 7.$

Hence, $x = 8$ and $y = 7$.

**36.** (*i*) Yes common difference = 12 – 6 = 18 – 12 = 6.
Next term of AP is 24 + 6 = 30.

(*ii*) Number of stitches in 10th row = 6 + 9 × 6 = 60.

(*iii*) $n$th term of AP is $6 + (n - 1) \times 6 = 6 + 6n - 6 = 6n$.

***Or***

Total number of stitches upto the 10th circular row

$$= \frac{10}{2}\{2 \times 6 + 9 \times 6\} = 5 \times (12 + 54)$$

$$= 5 \times 66 = 330.$$

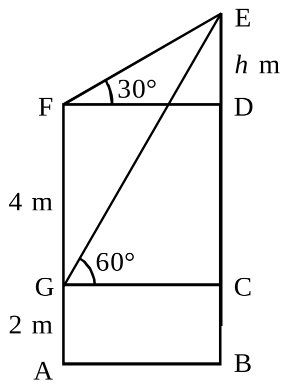
**37.** (*i*) Coordinates of L are (4, 7).

(*ii*) Coordinates of N are (12, 3).

(*iii*) Distance between L and O $= \sqrt{(4-5)^2 + (7-2)^2} = \sqrt{1 + 25} = \sqrt{26}$ units.

***Or***

Mid-point of line-segment LN $= \left(\frac{4+12}{2}, \frac{7+3}{2}\right) = (8, 5).$




**38.** (*i*) From $\Delta$EFD, $\frac{ED}{FD} = \tan 30° \Rightarrow \frac{h}{FD} = \frac{1}{\sqrt{3}}$ ...(1)

From $\Delta$EGC, $\frac{EC}{GC} = \tan 60° \Rightarrow \frac{h+4}{FD} = \sqrt{3}$ ...(2)

From (1) and (2), we get

$$\frac{h+4}{FD} \times \frac{FD}{h} = \sqrt{3} \times \frac{\sqrt{3}}{1}$$

$$\Rightarrow \frac{h+4}{h} = 3h = h + 4 \Rightarrow h = 2 \text{ metres.}$$

(*ii*) From $\Delta$EFD, $\frac{ED}{FD} = \tan 30° \Rightarrow \frac{2}{FD} = \frac{1}{\sqrt{3}} \Rightarrow$ FD $= 2\sqrt{3}$ metres.

So, horizontal distance of the balloon from the house is $2\sqrt{3}$ metres.

(*iii*) Now, EB = ED + DC + BC
= 2 + 4 + 2 = 8 metres.

So, height of the balloon from the ground is 8 metres.

***Or***

From $\Delta$EGC, $\frac{EC}{EG} = \sin 60° \Rightarrow \frac{2+4}{EG} = \frac{\sqrt{3}}{2} \Rightarrow$ EG $= \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3}$ metres.

Hence, distance of the balloon from window G is $\frac{12\sqrt{3}}{3}$ metres.